Curves: Curve of Archytas, Cycloid, Helical Spiral

Dr. Sarah's Differential Geometry

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as well as when I bring us back together.

1. Write down some items related to our explorations on the curve of Archytas. This could include sketches.

   - differential geometry of curve of Archytas [images]
   - `https://www.jstor.org/stable/24913477`
   - `https://mathcurve.com/courbes3d.gb/archytas/archytas.shtml`
   - `https://mathshistory.st-andrews.ac.uk/Biographies/Archytas/`
   - `https://library.appstate.edu/`
   - `https://library.appstate.edu/find-resources/databases/subject/mathematical-sciences`
     What is MathSciNet? Historically, mathematicians communicated by letters, during visits, or by reading each other's published articles or books once such means became available. For example, Marin Mersenne had approximately 200 correspondents. Some mathematical concepts were developed in parallel by mathematicians working in different areas of the world who were not aware of each other's progress. In an effort to increase the accessibility of mathematics research articles, reviews began appearing in print journals like Zentralblatt fur Mathematik, which originated in 1931, and Mathematical Reviews, which originated in 1940. Since the 1980s, electronic versions of these reviews have allowed researchers to search for publications. MathSciNet, the electronic version of Mathematical Reviews, is "the authoritative gateway to the literature of Mathematics" and currently contains over 4 million items.
   - `https://scholar.google.com/`
   - curve of Archytas $\alpha(t) = (\cos^2 t, \cos t \sin t, \sqrt{(1-\cos t)\cos t}\,)$

2. For the two curves:
   cycloid $\alpha(t) = (t + \sin t, 3 - \cos t, 0)$
   helical spiral $\alpha(t) = (3\cos t, 3\sin t, \ln t)$

   - Compute T by hand for each of these parametrizations (or parameterizations, if you prefer!) of the cycloid and helical spiral.
   - For each curve, discuss: Is T in the Frenet-Serret TNB frame defined everywhere in the domain $(.0000001, 2\pi)$? If not, specify any problem points and explain.
   - Discuss why is the cycloid interesting from a physics standpoint? You may use a web search. (We'll come back to helices later in terms of real-life applications.)
   - As time allows, we'll come back together to look at the Frenet-Serret frame in Maple. If you are finished before we come back together and have access, open the Maple file from the in-class items, video slides and more page at the top of ASULearn, copy the cycloid parametrization from below up top and explore. Sketch the curve and its frame at two different places on the curve. Distinguish between T, N and B.
   - Discuss: is B defined everywhere in this domain for both curves? Why or why not?