Rucker, TNB Frame, Curves Matching, and Spherical Epitrochoid

Dr. Sarah's Differential Geometry

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as I make my way around as well as when I bring us back together.

1. Sit in a group of 4 (if possible) and introduce yourselves to those sitting near you. What are their preferred first names?

2. For the phone cord curve, which of the following are true? Answer on pollev and review with your group.

   a) $\kappa = 1$ is constant but $\tau = \sin(s)$ varies
   b) $\tau = 1$ is constant but $\kappa = \sin(s)$ varies
   c) They are both constant
   d) They both vary
   e) Rudy Rucker said there is no way to know

3. For the rocker curve, which of the following are true? Answer on pollev and review with your group.

   a) $\kappa = 1$ is constant but $\tau = \sin(s)$ varies
   b) $\tau = 1$ is constant but $\kappa = \sin(s)$ varies
   c) This is a baseball stitch curve
   d) More than one of the above
   e) None of the above

4. To prove that the derivative of a unit vector $\vec{u}$ is perpendicular to the original, we...

   a) took the derivative of $\vec{u} \cdot \vec{u}$ and argued from there
   b) took the derivative of $\vec{u} \times \vec{u}$ and argued from there
   c) both of the above
   d) none of the above

5. As a review of the Frenet-Serret TNB Frame, discuss with your group:

   - How did we show $N \perp T$?
   - Why is B perpendicular to both T and N? Hint: consider how we defined B.
   - In what order did we explore the derivatives of the TNB frame and the Frenet-Serret equations?

6. Match the physical/geometric description in column 2 that is numbered 1–9 to a name in column 1 and a formula in column 3. If the description contains the word certain then clarify which vector or plane "certain" refers to.

torsion τ	1. how fast we are traveling along the curve in time	$\frac{\frac{T'(t)}{\lvert\alpha'(t)\rvert}}{\lvert\frac{T'(t)}{\lvert\alpha'(t)\rvert}\rvert}$
acceleration	2. how fast a certain tangent vector is changing in space and the deviation of the curve from being a straight line	$-\frac{B'(t)}{\lvert\alpha'(t)\rvert}\cdot N$
$T'(s)$	3. how far we are traveling along the curve in space	$(\frac{d^2\alpha^1}{dt^2},\frac{d^2\alpha^2}{dt^2},\frac{d^2\alpha^3}{dt^2})$
$B'(s)$	4. captures the movement of a certain tangent in time	$-\tau N$
arc length	5. how fast we are twisting out of a certain plane in space	κN
speed	6. captures the movement of a certain tangent in space	$\int \lvert\alpha'(t)\rvert dt$
$N'(s)$	7. captures the movement of a certain plane in space	$\lvert\frac{T'(t)}{\lvert\alpha'(t)\rvert}\rvert$
curvature κ	8. unit vector lies along the direction which the curve is currently bending in	$-\kappa T+\tau B$
normal N	9. turns away from a certain tangent and toward a certain binormal	$\sqrt{(\frac{d\alpha^1}{dt})^2+(\frac{d\alpha^2}{dt})^2+(\frac{d\alpha^3}{dt})^2}$

7. If you have access, search for information on the spherical epitrochoid and then explore the diffgeomsphericalepitrochoid.mw Maple file from the "in-class items, video slides and more" page at the top of ASULearn. If not, review from the video due today or discuss upcoming work like project 1.