Holey Donuts

Dr. Sarah's Differential Geometry

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as well as when I bring us back together.

1. Using the physical donut, also known as a torus, consider the tangent plane is at various points including:

   - a point at the very top of the donut when it is sitting on the table
   - a point at the outermost part (outside, halfway to the table) on the donut
   - a point at the innermost part (inside, halfway to the table) on the donut

   A normal to the surface is perpendicular to the tangent plane. Sketch some pictures of a unit normal at these various points on the donut in your notes.

2. Next, consider the circle that goes around the very top of the donut—the collection of all the highest points at the very top when the physical donut is sitting on the table. Compare the curvature vector $\vec{\kappa}_\alpha$ of points on this circle to the unit normal to the surface U along this circle. Which of the following is true? Discuss and then respond on `pollev.com/drsarah314`

   a) $\vec{\kappa}_\alpha$ is parallel to U so is not felt by the bug and this curve is a geodesic

   b) $\vec{\kappa}_\alpha$ is not parallel to U so is felt by the bug and this curve is not a geodesic

   c) other

3. What can we say about $\vec{\kappa}_n$, the normal curvature, the projection of $\vec{\kappa}_\alpha$ onto U for this top circle?

4. What can we say about $\vec{\kappa}_g$, the geodesic curvature, the remaining portion of the curve's curvature vector, if any, for this top circle?

5. Sketch the top circle on the donut, select one point on it, and sketch and label $\vec{\kappa}_\alpha$, U, $\vec{\kappa}_n$, and $\vec{\kappa}_g$ at this point. If any of the vectors are $\vec{0}$ then annotate on the side rather than trying to sketch and label the $\vec{0}$.

6. Look at a vertical circle of the donut, which runs from the outermost part of the donut to the very top to the inside part to the very bottom and back to the same point again. Sketch a picture of this on the donut, select one point on it, and sketch and label $\vec{\kappa}_\alpha$, U, $\vec{\kappa}_n$, and $\vec{\kappa}_g$ at this point. If any of the vectors are $\vec{0}$ then annotate on the side rather than trying to sketch and label the $\vec{0}$.

7. Is a vertical circle on the donut a geodesic?

8. Look at the outermost circle on the donut. Sketch a picture of this on the donut, select one point on it, and sketch and label $\vec{\kappa}_\alpha$, U, $\vec{\kappa}_n$, and $\vec{\kappa}_g$ at this point. If any of the vectors are $\vec{0}$ then annotate on the side rather than trying to sketch and label the $\vec{0}$.

9. Is this outermost circle on the donut a geodesic?

10. Look at the innermost circle on the donut. Sketch a picture of this on the donut, select one point on it, and sketch and label $\vec{\kappa}_\alpha$, U, $\vec{\kappa}_n$, and $\vec{\kappa}_g$ at this point. If any of the vectors are $\vec{0}$ then annotate on the side rather than trying to sketch and label the $\vec{0}$.

11. Is this innermost circle on the donut a geodesic?

12. Pull the string tightly to consider other geodesics on a donut.

13. Can a geodesic on the torus wrap around in interesting ways?

14. Which are true about curves and their speeds? Discuss and then respond on `pollev.com/drsarah314`

    a) If the speed of a curve is constant then it is a geodesic

    b) If the speed of a curve is not constant then it is not a geodesic

    c) both of the above

    d) none of the above

15. Discuss curves that are geodesics on the two-holed donut, a double torus, via symmetry arguments as well as $\vec{\kappa}_g = 0$. Sketch a picture of a geodesic that includes $\vec{\kappa}_\alpha$ parallel to U.

16. Discuss curves that are not geodesics on the two-holed donut via symmetry arguments as well as $\vec{\kappa}_g \neq 0$. Sketch a picture of a curve that isn't a geodesic and include $\vec{\kappa}_\alpha$, U, $\vec{\kappa}_n$, and $\vec{\kappa}_g$.

17. Are any of the curves shown in the following picture geodesics? Why or why not?