Dr. Sarah's Differential Geometry—Welcome!

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Discuss and review content from our prerequisite MAT 2130: Calculus with Analytic Geometry III via the questions below as you write your responses in your notes. You are free to use online sources too.

Try to help each other! Talk about them one at a time, as you go along, and keep track of any questions your group has. Feel free to ask me questions as I make my way around as well as when I bring us back together.

1. Sit in a group of 4 (if possible) and introduce yourselves to those sitting near you. What are their preferred first names?

2. Given $\vec{v} = \langle 2, 1, 6 \rangle$ and $\vec{w} = \langle 3, 1, -1 \rangle$, calculate $\vec{v} - \vec{w}$.

3. Write an equation for the line connecting the points $(2, 1, 6)$ and $(3, 1, -1)$ (same points as the endpoints of $\vec{v}$ and $\vec{w}$).

4. Given $\vec{v} = \langle 2, 1, 6 \rangle$ and $\vec{w} = \langle 3, 1, -1 \rangle$, calculate $\vec{v} \cdot \vec{w}$ and show the calculation.

5. What is a geometric or physical interpretation when $\vec{v} \cdot \vec{w} = 0$?

6. Find a tangent vector to the curve $\alpha(t) = (\cos(t), \sin(t), 5)$ at $t = 2\pi$.

7. Find the speed of the curve $\alpha(t) = (\cos(t), \sin(t), 5)$ at $t = 2\pi$.

8. What is the difference between tangent, velocity, and speed?

9. Find the arc length of the curve $\alpha(t) = (\cos(t), \sin(t), 5)$ from 0 to 2π and show the calculation.

10. Given $\vec{v} = \langle 2, 1, 6 \rangle$ and $\vec{w} = \langle 3, 1, -1 \rangle$, calculate $\vec{v} \times \vec{w}$ and show the calculation.

11. What is a geometric or physical interpretation of $\vec{v} \times \vec{w}$?

12. Find a vector that is perpendicular to both $\vec{v} = \langle 2, 1, 6 \rangle$ and $\vec{w} = \langle 3, 1, -1 \rangle$.

13. Using your last response, write an equation of the plane that both vectors lie in.

14. Find a unit vector pointing in the direction of $\langle 3, 1, -1 \rangle$.

15. Is $(1, 1, 5)$ on the curve $\alpha(t) = (\cos(t), \sin(t), 5)$? Why or why not?

16. Let $f(x, y) = 1 + 2x\sqrt{y}$. Find the directional derivative at the point $(3, 4)$ in the direction of $\vec{v} = \langle 4, -3 \rangle$.

17. What is a geometric or physical interpretation of $\arccos(\dfrac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|})$?
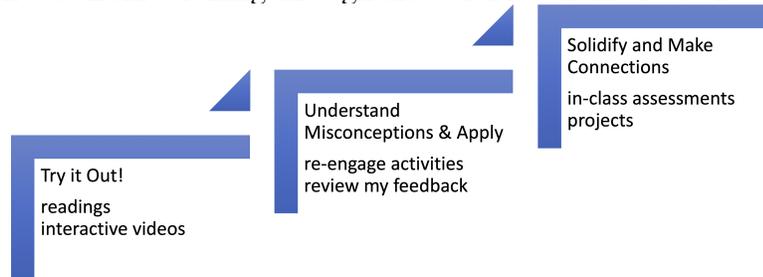
If your group is finished discussing and responding to the above before I bring us back together, then

- Consider why is a line shorter than any other curve $\alpha(t)$ between 2 points in Euclidean geometry?

- Chat to get to know your classmates better and build community!

Differential Geometry FAQ and Engagement—Optimize your Success and Understanding!

- Where can I find in-class and out-of-class activities?

  On our ASULearn! The sections organized by due dates have completion activities and the "in-class items, video slides and more" link in the top section of ASULearn on need help?/course info has in-class worksheets and more. Turn all items to Done by their due date. Major assignments, including projects and reflections, are also turned in on ASULearn. The ASULearn components work best from scrolling through the activities themselves on a computer.

Solidify and Make Connections

in-class assessments
projects

Understand Misconceptions & Apply

re-engage activities
review my feedback

Try it Out!

readings
interactive videos

- How do I contact you outside of class?

  need help from me, your classmates, or tech support? at the top of ASULearn (not e-mail that gets buried!) The Zoom link there is for office hours
  Sunday, Tuesday, Thursday 7-7:45pm
  Monday, Wednesday 12:30–1:30pm

  These are already over the number of hours officially required but I will sometimes be able to add an extra hour Friday 2–3pm—check the need help forum for updates. If you can't make Zoom, select the dropdown item listing only you and I to contact me privately, or the whole class to send a message to everyone! Please use a salutation of Dr. Sarah, my preferred name, in communications with me. I strive to answer individual questions at least once a day, including the weekends, although I may respond within class. I prefer that you use real-time hours as it is easier to discuss material in person.

- What should I do if I don't understand content or something about the course?

  If it is differential geometry content, come see me in office hours and ask me questions during class.

  If it is material from Calculus With Analytic Geometry III, the prerequisite, or earlier courses, it will be helpful to review to make sure you understand the concepts and know how to do a computational example as we use them in class. This includes
    - acceleration and velocity
    - arc length
    - cross product
    - curvature
    - cylindrical and spherical coordinates
    - derivative of a function of one variable whose range is in $\mathbb{R}^3$, i.e. $(x(t), y(t), z(t))$
    - directional derivative
    - dot product
    - equation of a line in 3-space
    - equation of a plane
    - fundamental theorem of calculus
    - gradient
    - magnitude, norm or length of a vector
    - multivariable chain rule
    - normal vector
    - parameterizations of curves and surfaces
    - partial derivatives of a multivariable function i.e. $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for $f(x, y)$

- speed
- surface area
- tangential and normal components of a vector like acceleration
- tangent line
- tangent plane
- tangent vector
- volume

If it is material from Introduction to Linear Algebra, the co-requisite, it would be helpful to pay special attention as these concepts as they come up in 2240 and in differential geometry, or review them if you have already taken the class:

- addition of matrices
- a_{ij} matrix notation
- basis of a space
- determinant of a matrix
- dimension of a space
- eigenvalue λ from $A\vec{x} = \lambda\vec{x}$
- linear combination of vectors
- multiplication of matrices, including $A\vec{v}$ and AB
- inverse of a 2x2 matrix
- span of vectors
- symmetric matrix $A^T = A$
- transpose of a matrix $A^T = (a_{ji})$ for $A = (a_{ij})$

If it is an ASULearn activity, I have instructions inside each activity link on ASULearn, at the top. Ask questions inside and outside of class. Access (or re-watch) the course intro interactive video, which explains many components. My course design is intentional and based on best practices from the scholarship of teaching and learning including *Make it Stick: The Science of Successful Learning*. Depending on your prior experiences, it may take some getting used to—I'm here to help you!

Tentative Calendar

While some items have strict deadlines, there is still flexibility built in and multiple pathways for success—videos have multiple chances to succeed and projects can be completed ahead plus there is a revision opportunity for one of the first three projects and one in-class assessment. Attempt readings and videos for completion and take video notes by the listed date when possible as the material builds on itself. Some days are lighter than others and it will help you to progress on upcoming activities in advance, especially major assignments.

	Class Monday	Between Classes (by just before 3pm Wed.)	Class Wednesday	Between Classes (by just before 3pm Monday)
1/17		obtain rental book from bookstore	review 2130	-class intro interactive video - read "Curves" -read 1.1 pp. 1–7 -lines and Maple intro interactive video -download or access Maple -re-engage 1
1/22-1/24	curvature osculating circle parabola and line	-read 1.1 pp. 8–14 -tractrix interactive video -add ASULearn profile pic -Zoom update & profile pic -get to know posting -read the syllabus	arc length and speed comparing and contrasting curves	-read 1.2 pp. 14–17 -s, T and physical attributes interactive video -practice submitting PDF -re-engage 2
1/29–1/31	s, T, velocity, speed, acceleration, jerk helix computations	-read 1.3 pp. 17–19 -TNB 1 interactive video -choice of curve for Project 1	TNB curve of Archytas cycloid and spiral	-read "How Flies Fly" -read 1.3 pp. 19–20 -TNB 2 interactive video -re-engage 3

2/5–2/7	TNB spherical epitrochoid matching activity	-read 1.3 21–25 -curvature and torsion implications 1 interactive video	curvature and torsion Darboux vector fundamental theorem of space curves	-Project 1: research, investigate and present a curve -re-engage 4
2/12–2/14	Project 1 presentations	-read 1.5 pp. 34–35 -curvature and torsion implications 2 interactive video -begin assessment guide	curvature and torsion helix and strake	-prepare for in-class curves assessment -re-engage 5 -complete any open items
2/19–2/21	in-class curves assessment	-surfaces, geodesics and coverings interactive video -read pp. 247–250	covering geodesics cone	-read pp. 67–68, 77–82, 209 -coordinates and geodesic curvature interactive video -re-engage 6
2/26–2/28	geodesics sphere spherical coordinates	-read pp. 70–76, 212 -speed of a geodesic interactive video	geodesics round donut double torus	-read "Surfaces" -first fundamental form interactive video -choose surface for Project 2 -re-engage 7
3/4–3/6	geodesics metric form flat and round donuts	-read pp. 83–87 -shape operator interactive video	shape operator mystery surface round donut Catalan surface	-read pp. 88–91, 91–96, 107–108, 111–114, 123–124 -II and Gauss's Theorema Egregium interactive video -re-engage 8
3/18–3/20	π-day Gauss and mean curvature	-read p. 164 -surface area interactive video	surface area matching activity	-read pp. 275–277, 289–292 -Gauss Bonnet video -re-engage 9
3/25–3/27	Gauss Bonnet	-Project 2: research, investigate and present a surface	Project 2 presentations	-read pp. 226–235 -surfaces not embedded interactive video -begin assessment guide -re-engage 10
4/1–4/3	surfaces not in $\mathbb{R}^3$ Klein bottles hyperbolic	- prepare for in-class surfaces assessment -re-engage 11 -complete any open items	in-class surfaces assessment	-read pp. 397–416 -geodesic equations, tensors and spacetime interactive video
4/8–4/10	curvatures Christoffel symbols Γ^a_{bc} Poincaré upper-half plane	-read "How to Create Your Own Universe in Three Easy Steps" -Minkowski spacetime and Christoffel computations interactive video -choose metric for Project 3	spacetime metric form research	-read pp. 416–430 -wormhole metric, curvatures and relativity interactive video -re-engage 12
4/15–4/17	curvatures and Γ^a_{bc} Brenton universe discuss final project	-begin final project	work on project 3 or final project	-Project 3: research, investigate and present a metric form -re-engage 13
4/22–4/24	Project 3 presentations	-general relativity and the field equations interactive video -read "Relativity"	relativity concluding activities	-course survey -course evaluations -re-engage 14
4/29–5/1	work on final project or optional revisions	-complete any open items	share final project idea or title	-final project video
5/6	turn in video presentation on ASULearn by the beginning of our 2pm assigned time during finals during the assigned time, conduct video project peer review and self-evaluation (optional) revise and reflect on one in-class assessment, one of the first three projects			