Sphere and Spherical Coordinates

Dr. Sarah's Differential Geometry

Welcoming Environment: Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as well as when I bring us back together.

1. Consider symmetry arguments on the physical sphere. Is a latitude (horizontal circle) on a sphere always a geodesic? Discuss and then respond on `pollev.com/drsarah314`
   a) yes and I have a good reason why
   b) yes but I am unsure of why
   c) no but I am unsure of why not
   d) no and I have a good reason why not

2. We can't flatten a sphere with a C^2 isometry (a map that preserves distance, angles and more with continuous first and second derivatives), so we can't use paper covering arguments like we did for the cylinder and cone. The sphere is its own cover. However, we can still use symmetry arguments as well as string pulled tightly to consider geodesic paths between 2 points. Using the physical sphere in front of you, answer the following questions, sketch pictures in your notes and annotate your responses:
   - How many differently shaped geodesics can you find?
   - How many geodesics join 2 points?
   - Can a geodesic ever intersect itself? Why or why not?
   - Is shortest distance by pulling the string tight on the sphere always part of an intrinsically straight geodesic on the sphere?
   - Is intrinsically straight on the sphere always shortest distance on the sphere?

3. Once we have a parametrization, we or Maple will be computing a unit normal to a surface via $U = \dfrac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$. Even without a parametrization, we can still get an intuitive sense of the normal direction as it's normal to the surface and normal to the tangent plane!

   So... use the stiff paper I've provided to place the tangent plane to the sphere at approximately the north pole and place a pen or pencil so that it creates a normal to the surface and normal to the tangent plane at the north pole. First, sketch a picture in your notes.

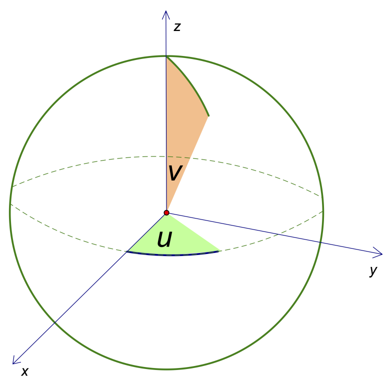
4. Next, consider the vector from the center of the sphere $(0, 0, 0)$ to the north pole. Is there any kind of relationship between that vector and the pen or pencil normal at the north pole?

5. More generally, how does a surface normal at a given point on the sphere relate to a vector that starts at the center and ends at that point on the sphere? Discuss and then respond on `pollev.com/drsarah314`
   a) there is no relationship between them
   b) they are perpendicular
   c) they are parallel
   d) other

6. In your notes, sketch a non-equator latitude $\alpha(t)$ on a sphere, select a point on it, and add the following to your sketch. For ease of sketching, assume the latitude has radius 1 so that the length of the curvature vector, the reciprocal of the radius, is the same, although I'll show you one where that is not so in the re-engage.
   - $\vec{\kappa}_\alpha$, the curvature vector to the latitude curve, just like back in the curves section
   - $\vec{\kappa}_n$, the normal curvature, the projection of $\vec{\kappa}_\alpha$ onto a unit surface normal
   - $\vec{\kappa}_g$, the geodesic curvature, what is leftover to be felt by the bug in the tangent plane

   You can use the physical sphere to help solidify the visualization.

7. There are different coordinate systems for a sphere. In the upcoming interactive video, we'll look at geographic coordinates on a sphere of radius r. Here though, let's look at a sphere of radius 1 in spherical coordinates $\mathbf{x}(u, v) = (\cos u \sin v, \sin u \sin v, \cos v)$, which you made use of in Calculus with Analytic Geometry III, probably with $u = \theta$ from 0 to 2π and $v = \phi$ from 0 to π.



   To investigate what does a u coordinate curve look like on the sphere, try selecting a revealing or especially simple value of a constant v to help you explore that specific u coordinate curve. Then consider a number of u coordinate curves. What can we say about the u coordinate curves? Sketch some examples in your notes and annotate.

8. Is a u coordinate curve on the sphere always part of a geodesic? If so, say yes. If not, provide a counterexample and then discuss which ones are, if any?

   Respond on `pollev.com/drsarah314`

   a) yes and I have a good reason why
   b) yes but I am unsure of why
   c) no but I am unsure of why not
   d) no and I have a good reason why not

9. What does a v coordinate curve look like on the sphere where u is constant?

10. Is a v coordinate curve on the sphere always part of a geodesic? If so, say yes. If not, provide a counterexample and then discuss which ones are, if any?

11. If time remains and you have access, open the Maple file diffgeomsphere.mw under today's date from the "in-class listings, video slides and more" link at the top of ASULearn and input spherical coordinates with a radius $\neq 1$ of your choice and explore the commands and outputs for various curves on the spherical coordinate sphere.