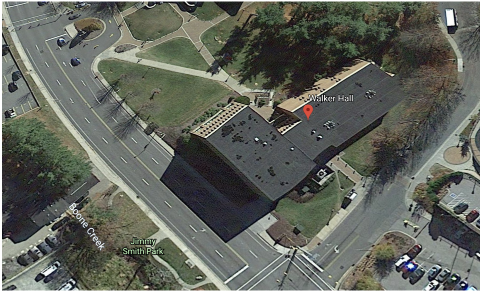
Surfaces

Each point has a neighborhood that locally looks like a plane.

manifolds and surfaces versus orbifolds

Google Earth



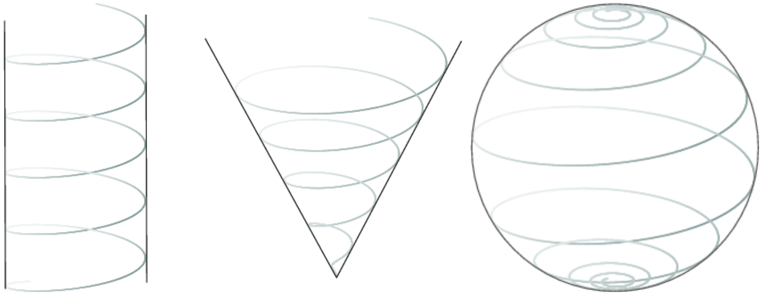
Google Earth

sphere versus ball

Surfaces: Visual and Tactile Perspectives

- At a very early age, children develop a very rich ‘visual intelligence’ in terms of perception and experiences. They have questions and lots of these questions and explorations can be connected to geometry if we use the right types of physical and visual presentations... the precise vocabulary is... differential geometry and differential topology... we should connect with these abilities. (Whiteley, 1999)
- Open your eyes and hands to surfaces in our world—aesthetics and applicability

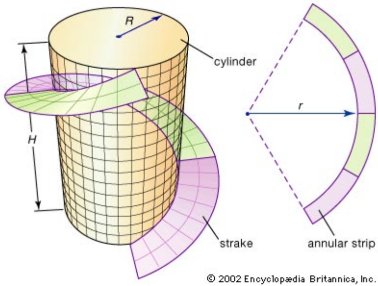
Curves→Surfaces



http://www.tankonyvtar.hu/en/tartalom/tamop425/0038_matematika_Miklos_Hoffmann-Topology_and_differential_geometry/images/csavar_hengeres_kupos_cdr.png

helix, loxodrome or rhumb Line

Strake Parametrization



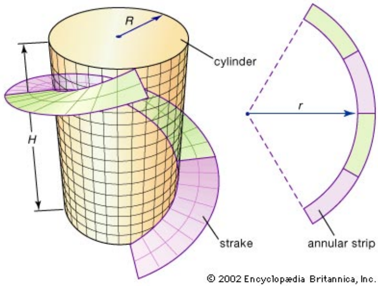
https://cdn.britannica.com/22/70822-004-B85BF4BD/

strake-strip-dimensions-cylinder-contour-Techniques-differential.jpg

inner helix = $(3.75 \cos t, 3.75 \sin t, \frac{31.5}{2\pi} t)$

outer helix = $(4.35 \cos t, 4.35 \sin t, \frac{31.5}{2\pi} t)$

Strake Parametrization



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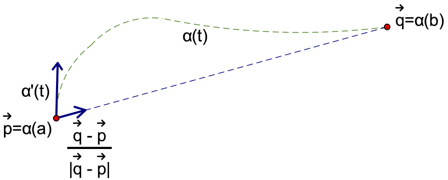
inner helix = $(3.75\cos t, 3.75\sin t, \frac{31.5}{2\pi}t)$
outer helix = $(4.35\cos t, 4.35\sin t, \frac{31.5}{2\pi}t)$
$X(r,t) = (r\cos(t), r\sin(t), \frac{31.5}{2\pi}t)$
$r \in (3.75, 4.35), t \in (0, 2\pi)$

Geodesic—Intrinsically Straight Path

SURFACE TENSION

$l(t) = \vec{p} + t(\vec{q} - \vec{p})$ Geodesic in Plane



$$|\vec{q} - \vec{p}| = \frac{|\vec{q}-\vec{p}|^2}{|\vec{q}-\vec{p}|} = (\vec{q} - \vec{p}) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}$$

$$= \vec{q} \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|} - \vec{p} \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|} = \alpha(b) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|} - \alpha(a) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}$$

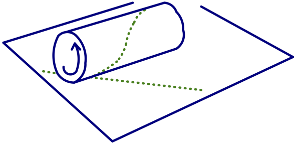
$$= \int_a^b (\alpha(t) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|})' dt = \int_a^b \alpha'(t) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|} + \alpha(t) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}' dt$$

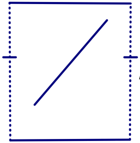
$$= \int_a^b \alpha'(t) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|} dt$$

$$= \int_a^b |\alpha'(t)| |\frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}| \cos\theta(t) dt$$

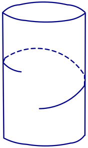
$$\leq \int_a^b |\alpha'(t)| |\frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}| dt = \int_a^b |\alpha'(t)| dt$$

(Intrinsically Straight) Geodesics on a Cylinder





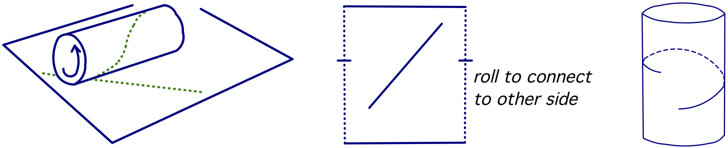
roll to connect to other side



- symmetry—toy car steering or our feet
- rolling arguments (covering arguments in general): draw a straight line on a piece of paper, roll up and connect the edges to form a cylinder and see what the line becomes

1. Can a geodesic ever intersect itself?
2. How many differently shaped geodesics can you find? What do they look like?
3. Is straight always shortest distance?
4. Is shortest distance always straight?
5. How many geodesics join 2 points?

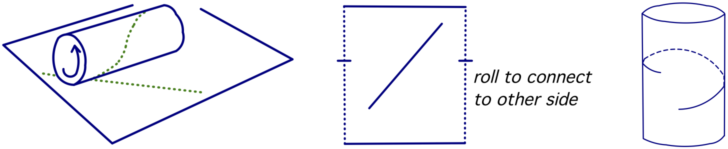
Geodesics on a Cylinder



- symmetry—toy car steering or our feet
- rolling arguments (covering arguments in general)

1. Geodesic ever intersect itself?

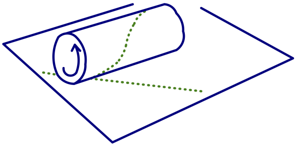
Geodesics on a Cylinder

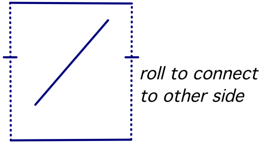


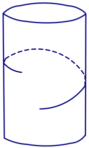
- symmetry—toy car steering or our feet
- rolling arguments (covering arguments in general)

1. Geodesic ever intersect itself? Yes. horizontal circle
2. Shapes?

Geodesics on a Cylinder



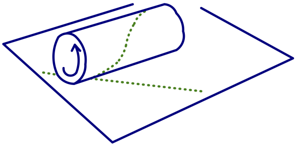


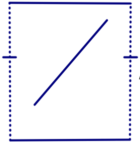


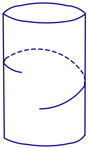
- symmetry—toy car steering or our feet
- rolling arguments (covering arguments in general)

1. Geodesic ever intersect itself? Yes. horizontal circle
2. Shapes? vertical lines, horizontal circles, helices (constant angle is made with the z-axis because it is a straight line on the unrolled cylinder)
3. Straight always shortest distance?

Geodesics on a Cylinder







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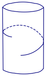
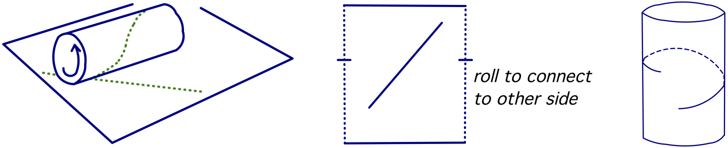


3. Straight always shortest distance? No.
4. Shortest distance always straight?

Geodesics on a Cylinder



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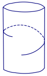
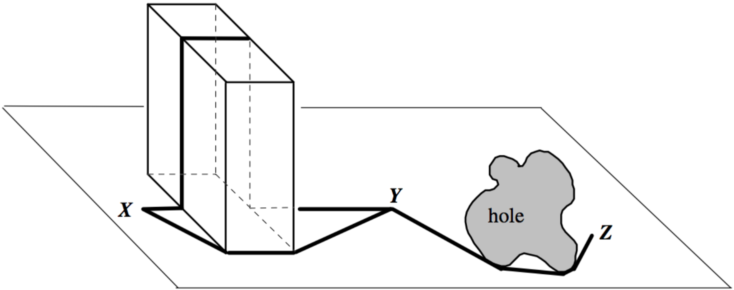


3. Straight always shortest distance? No.
4. Shortest distance always straight? Yes. shortest on cylinder is shortest on covering & hence intrinsically straight on both

Geodesics on a Smooth Surface in the C^2 Sense

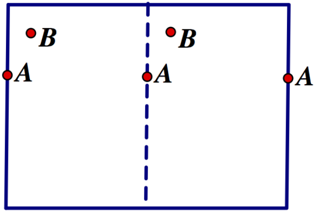
- smooth surface has geodesics that are locally shortest distance
- smooth surface that is geodesically complete can have points be joined by a shortest distance geodesic



David Henderson: http://pi.math.cornell.edu/~dwh/papers/EB-DG/EB-DG-web.htm

Geodesics on a Cylinder

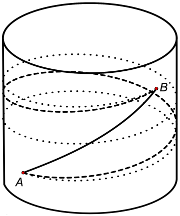
5. How many geodesics join 2 points? A 2-sheeted covering:



- Fold a paper in half vertically so you have 2 equal regions
- Label point A on each edge at the same height (3 As)
- Choose Bs not on the same vertical or horizontal line as A
- Draw a line between every A and every B. Marker is best.
- Roll the sheet up so As match & examine the geodesics

Geodesics on a Cylinder

5. How many geodesics join 2 points?

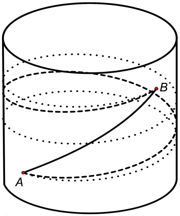


3 of 6 geodesics from a 3-sheeted covering of the cylinder.

“Adapted from Parameterization of tubular surfaces on the cylinder” by Toon Huysmans and Jan Sijbers

Geodesics on a Cylinder

5. How many geodesics join 2 points?



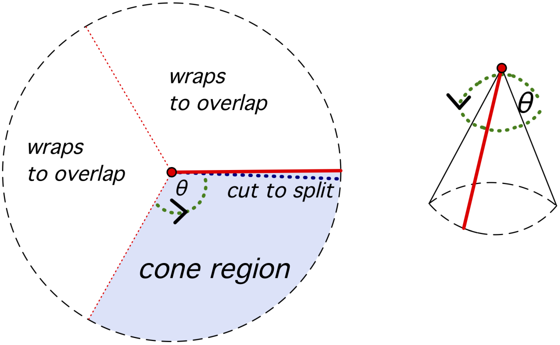
3 of 6 geodesics from a 3-sheeted covering of the cylinder.

"Adapted from Parameterization of tubular surfaces on the cylinder" by Toon Huysmans and Jan Sijbers

- horizontal points: 1
  they are part of the same geodesic circle, aside from # times it overlaps or goes around front or back
- non-horizontal points—keep adding sheets to the covering: ∞ (countably)

Geodesics on a Cone

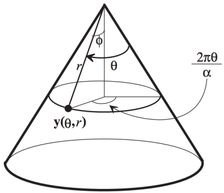
$0 <$ cone angle $< 2\pi$ variable cone

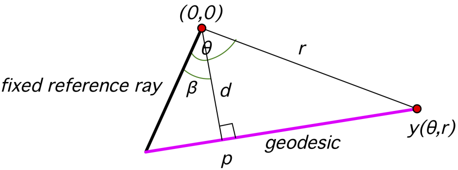


$\frac{2\pi}{3}$ *cone*

We can vary the angle by changing the cone region before we wrap the rest around (it doesn't have to fit evenly into 2π)

Geodesic Equation on a Cone





geodesic equation $r = d\sec(\theta - \beta)$