

# Contents

# MAT 4140: Differential Geometry

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## 1.1 Course Goals and Objectives

Catalog: This is an introductory course in the differential geometry of curves and surfaces in space, presenting both theoretical and computational components, intrinsic and extrinsic viewpoints, and numerous applications. The geometry of space-time will also be considered. Prerequisite: MAT 2130. Corequisite: MAT 2240.

This course includes:

- Geometry of curves in space, including Frenet formulas
- Theory of surfaces, including curvature, geodesics, and metrics
- Geometry of space-time and applications to general relativity

- To develop geometric skills and 3D spatial visualization skills.
- To develop a greater appreciation for connections between various disciplines of mathematics, including geometry, linear algebra, analysis, and differential equations, along with an introduction to these subjects as they apply to differential geometry.
- To understand the importance of differential geometry in various scientific fields, including physics.
- To practice critical and creative thinking and to communicate effectively with your peers.

## 1.2 Required Resources

- *Differential Geometry and Its Applications*, John Oprea, 3rd edition, MAA Press: An Imprint of the American Mathematical Society, ISBN: 978-1-4704-5050-2, 2019.
- scanning handwritten work: the course is designed so that you'll collate responses, by-hand work, and Maple work for projects into a full size multipage PDF for submission in ASULearn. Electronically, you can append a Maple PDF you create to the end of a handwritten or professionally typeset PDF, like Preview on a Mac or PDFsam on a PC. Alternatively, if you are printing physically to paper, you can print your Maple work and then append it to the end of the handwritten work and then scan it. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help: <https://library.appstate.edu/services-search/print-zone-tech-help>.
- reliable access to technology, software, and high speed connectivity: The work you're going to do outside of class you'll be accessing through your internet connection with a device, so it will be really important to have access to a dependable high-speed internet connection and a good computer that can run everything we'll need. The software is free, including Maple—any student with a valid Appstate email address can access Maple on their own computer or on campus computers. For optional office hours, we'll use Zoom videoconference software. Flexible browsers that will play common media formats from various sources such as from webpages, Google Drive, YouTube, and ASULearn, including interactive videos, are also something we'll use. You may need some flexibility in browsers so that if one browser is incompatible, you can try another. ASULearn components work best from scrolling through the activities themselves on a computer—the Moodle mobile app does not always show everything as designed, both for visibility and for due dates. Hybrid course students are expected to have or acquire proficient computer skills and to resolve their own technology problems related to computers or internet access to turn things in

before deadlines. Work early and leave yourself enough time before any strict deadlines to allow for technical issues that may arise. For example, ASULearn will typically be undergoing monthly system maintenance on the last Thursday of each month. <https://asulearn.appstate.edu/?redirect=0> announces downtime and <http://Support.appstate.edu> and <https://confluence.appstate.edu/display/ATKB/Appalachian+Technology+Knowledge+Base> can help.

### 1.3 Assignment Types, Grades, and Policies

- **Effective ASULearn Engagement 30%**

ASULearn completion activities may be ones where you can manually mark the activity as completed or are earned for a good faith effort when you access an activity or receive a proficient grade by a deadline. Attempt items for completion and take interactive video notes by the listed date when possible as the material builds on itself. Generally, the percentage of completed activities determines the overall engagement grade. While many items have strict deadlines, there is flexibility built in and multiple pathways for success—interactive videos have multiple chances to succeed, many ASULearn items can be completed ahead, plus, to accommodate for emergencies, the lowest 3 completion assignments are dropped. In addition, evidence of deep and conscientious engagement in the activities is a part of the grade, which may be adjusted at the end of the semester to reflect it. ASULearn provides detailed reports. Activities may include web pages, PDFs, files, interactive videos, surveys, forums, turn-ins, re-engages, readings, begin in-class assessment guide, select a topic, and peer review. Readings include those from the book as well as the following short readings: *Curves* and *Surfaces*, both by Doğan Çömez, myself and Jill Thomley, *How Flies Fly: Kappatau Space Curves* by Rudy Rucker, *How to Create Your Own Universe in Three Easy Steps* by Lawrence Brenton, and *Relativity* by David Brink.

- **Projects 40%**

There are 4 projects over the course of the semester, which include research, investigations, and presentations to your classmates. The fourth project is during our assigned time at finals: as listed at the registrar “An instructor may only change the examination schedule for a course with the approval of the department chairperson and the dean.” Your work must be turned in on or before the due date at the beginning of class\*. To accommodate issues that may arise, there is a revision opportunity for one of the first three projects.

- **In-class Assessments 20%**

There are two timed in-class assessments designed to help you solidify and make connections. To encourage these as a learning experience, accommodate for emergencies, and help solidify your knowledge, there is a revision opportunity on one of them. Otherwise, no make ups allowed\*.

- **Effective Class Engagement 10%**

When I was designing this hybrid class, ASU’s Center for Excellence in Teaching and Learning for Student Success designated that “Face-to face component is not a lecture but provides time for discussion, demonstrations, problem-solving, and higher-level thinking and collaborative activities. Class time is used to apply course content in ways that can only be accomplished when everyone is together in the same place.”

<https://cae.appstate.edu/teaching-learning/course-delivery-options>. Effective class engagement is essential to our course integrity and attendance is required at ALL such classes, with the exception of legitimate or excused absences.

If you can’t attend in person but can attend electronically, ask someone in the class to bring you in at their table and let me know you are there—this would not count as an absence. To allow for weather issues, emergencies, sickness or other absences, there will be no penalty for missing less than 3 classes. If the university cancels classes or changes them, continue progressing on the work due on ASULearn and check ASULearn for any updated info, which may include plans for the missed class such as Zoom meetings or individual activities.

If you expect to miss more than 10% of classes due to university sponsored activities or other reasons then I advise you to drop the course. Any student who wants to obtain an “excused absence” for less than 10%

of classes must meet certain responsibilities, including contacting me on the ASULearn forum in advance when possible, providing official documentation, and making up any possible work in advance.

It will be useful if you bring a computer, tablet, or phone to classes that can access webpages—if not, you can still participate in other ways.

ASU prepares students to employ various modes of communication that can help communities reach consensus or respectful disagreement: successful communicators interact effectively with people of both similar and different experiences and values and in this class you will practice oral and written communication during class by interacting with your peers and me. Regardless of gender, political party, race, religion, sexuality, or more this class is to be a welcoming environment, and so I want you to be sensitive and respectful to each other in upcoming discussions. Keep it a safe place to express meaningful ideas and opinions. Actively listen to others and encourage everyone to participate. Part of the welcoming environment is to keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter. Performing activities that detract from the welcoming environment or distract your neighbors or me will result in a lowered grade. Asking or answering related and thought-provoking questions, coming up with creative ways of thinking about the material, and explaining the material to others are some examples of positive class engagement that will increase your grade.

\* Accommodations in the determination of your final grade may be made for extenuating circumstances that are officially documented to prevent you from completing work early/on time.

The grading scale is:  $A \geq 93$ ;  $90 \leq A- < 93$ ;  $87 \leq B+ < 90$ ...

## 1.4 Tentative Calendar

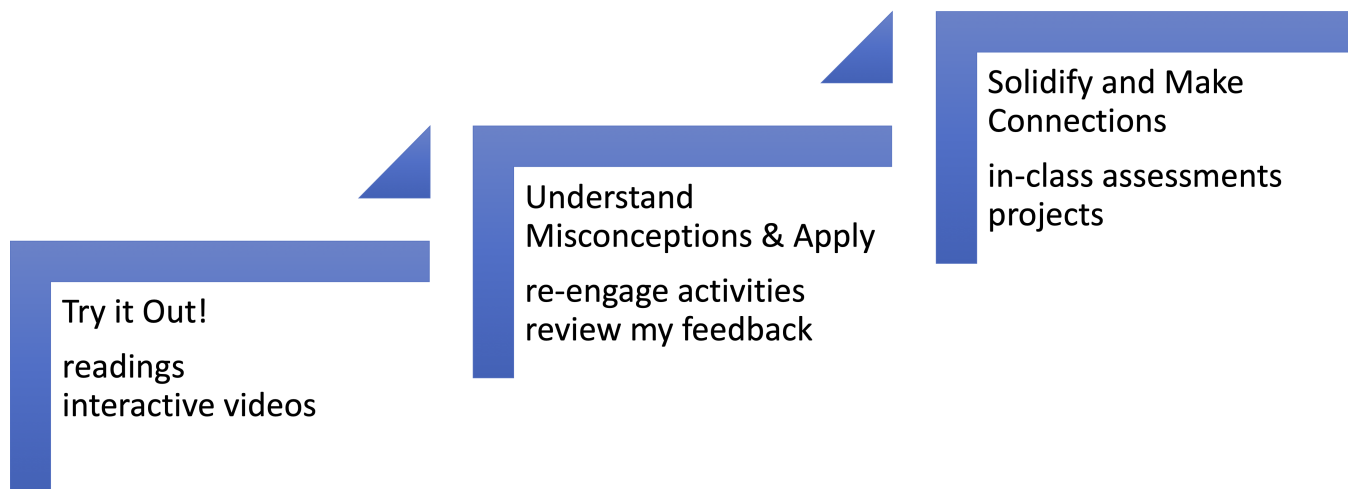
This class has two of our three credit hours as in-person meetings. You'll have daily work between our classes but you have the flexibility to work ahead to meet the deadlines if that is better for your schedule: plan to spend 3.5–5 hours between classes, on average, as per the University-wide Statement on Student Engagement with Courses. Our hybrid class is officially designed by the registrar and scheduled by the university for our third hour to be a part of the activities between classes.

Details for assignments and in-class activities and any changes are on <https://www.apstate.edu/~greenwaldsj/class/4140/s24.html> and ASULearn.

	Class Monday	Between Classes (by just before 3pm Wed.)	Class Wednesday	Between Classes (by just before 3pm Monday)
1/17		obtain rental book from bookstore	review 2130	-class intro interactive video -read "Curves" -read 1.1 pp. 1–7 -lines and Maple intro interactive video -download or access Maple -re-engage 1
1/22– 1/24	curvature osculating circle parabola and line	-read 1.1 pp. 8–14 -tractrix interactive video -add ASULearn profile pic -Zoom update & profile pic -get to know posting -read the syllabus	arc length and speed comparing and contrasting curves	-read 1.2 pp. 14–17 - $s$ , $T$ and physical attributes interactive video -practice submitting PDF -re-engage 2
1/29– 1/31	$s$ , $T$ , velocity, speed, acceleration, jerk helix computations	-read 1.3 pp. 17–19 -TNB 1 interactive video - <a href="#">choice of curve for Project 1</a>	TNB curve of Archytas cycloid and spiral	-read "How Flies Fly" -read 1.3 pp. 19–20 -TNB 2 interactive video -re-engage 3

2/5– 2/7	TNB spherical epitrochoid matching activity	-read 1.3 21–25 -curvature and torsion impli- cations 1 interactive video	curvature and torsion Darboux vector fundamental theorem of space curves	- <a href="#">Project 1: research, investi- gate and present a curve</a> -re-engage 4
2/12– 2/14	<a href="#">Project 1 presenta- tions</a>	-read 1.5 pp. 34–35 -curvature and torsion impli- cations 2 interactive video - <a href="#">begin assessment guide</a>	curvature and torsion helix and strake	-prepare for in-class curves assessment -re-engage 5 - <a href="#">complete any open items</a>
2/19– 2/21	<a href="#">in-class curves assess- ment</a>	-surfaces, geodesics and cov- erings interactive video -read pp. 247–250	covering geodesics cone	-read pp. 67–68, 77–82, 209 -coordinates and geodesic curvature interactive video -re-engage 6
2/26– 2/28	geodesics sphere spherical coordinates	-read pp. 70–76, 212 -speed of a geodesic interac- tive video	geodesics round donut double torus	-read “Surfaces” -first fundamental form in- teractive video - <a href="#">choose surface for Project 2</a> -re-engage 7
3/4– 3/6	geodesics metric form flat and round donuts	-read pp. 83–87 -shape operator interactive video	shape operator mystery surface round donut Catalan surface	-read pp. 88–91, 91–96, 107– 108, 111–114, 123–124 -II and Gauss’s Theorema Egregium interactive video -re-engage 8
3/18– 3/20	$\pi$ -day Gauss and mean cur- vature	-read p. 164 -surface area interactive video	surface area matching activity	-read pp. 275–277, 289–292 -Gauss Bonnet video -re-engage 9
3/25– 3/27	Gauss Bonnet	- <a href="#">Project 2: research, investi- gate and present a surface</a>	<a href="#">Project 2 presenta- tions</a>	-read pp. 226–235 -surfaces not embedded in- teractive video - <a href="#">begin assessment guide</a> -re-engage 10
4/1– 4/3	surfaces not in $\mathbb{R}^3$ Klein bottles hyperbolic	- prepare for in-class surfaces assessment -re-engage 11 - <a href="#">complete any open items</a>	<a href="#">in-class surfaces as- sessment</a>	-read pp. 397–416 -geodesic equations, tensors and spacetime interactive video
4/8– 4/10	curvatures Christoffel symbols $\Gamma_{bc}^a$ Poincaré upper-half plane	-read “How to Create Your Own Universe in Three Easy Steps” -Minkowski spacetime and Christoffel computations in- teractive video - <a href="#">choose metric for Project 3</a>	spacetime metric form research	-read pp. 416–430 -wormhole metric, curva- tures and relativity interac- tive video -re-engage 12
4/15– 4/17	curvatures and $\Gamma_{bc}^a$ Brenton universe discuss final project	- <a href="#">begin final project</a>	work on project 3 or fi- nal project	- <a href="#">Project 3: research, inves- tigate and present a metric form</a> -re-engage 13
4/22– 4/24	<a href="#">Project 3 presenta- tions</a>	-general relativity and the field equations interactive video -read “Relativity”	relativity concluding activities	-course survey -course evaluations -re-engage 14

4/29– 5/1	work on final project or optional revisions	-complete any open items	share final project idea or title	-final project video
5/6	turn in video presentation on ASULearn by the beginning of our 2pm assigned time during finals during the assigned time, conduct video project peer review and self-evaluation (optional) revise and reflect on one in-class assessment, one of the first three projects			



## 1.5 Course Communication, Office Hours, Where to Get Help, and Additional Policies

- I encourage you to talk to me and your classmates often in and out of class. Outside of class, I can help in office hours on Zoom, internet allowing, Sunday, Tuesday, and Thursday 7–7:45pm and Monday and Wednesday 12:30–1:30pm via the link in the Need Help Forum on ASULearn from <https://asulearn.appstate.edu/course/view.php?id=165635>

You can come in to Zoom to work on future assignments, working along and asking questions as they arise, or you can ask me questions about items you already turned in, ask me questions related to course content or structure, or ask me to go over a concept. Keep me informed about any problems and I also want to hear when things are going well! We can both write on a whiteboard and share our screens. You do not need to make an appointment to meet with me in Zoom, just drop by and I am happy to help! During class and in online communication, use standards of professionalism and collegial communication. This course is to be an environment in which everyone feels comfortable asking questions, making mistakes, offering good guesses and ideas, and is respectful to one another.

Please use a salutation of Dr. Sarah, my preferred name, in communications with me. If you can't make it to Zoom, you can contact me with a professional message on the Need Help Forum on ASULearn, which I'll try to answer once a day, including the weekends. If you haven't heard back from me yet, only message again if you are addressing a new and separate issue or retracting a previous message. Private conversations with me should occur during office hours or via the forum, not during class time. In Zoom hours, I can set up individual breakout rooms for private conversation with me. Except for extreme emergencies, all private written communication must be handled through your private forum, with just you and I, on ASULearn rather than e-mail. I prefer that you use the Zoom hours as it is easier to discuss material in person.

- The Conference Board of the Mathematical Sciences (CBMS) published a statement titled “Active Learning in Post-Secondary Mathematics Education” about the importance of “classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking” and our classroom is modeled after that. The purpose of engagement is to learn and practice course content and learning goals, and develop critical thinking and problem-solving skills. Making mistakes is integral to the learning process—the key is to try to continue to engage rather than give up—and this course is to be an environment in which you ask questions and offer good guesses. It is on purpose that

there are problems that don't look exactly like what we did previously in order to provide you with rich settings to explore in order to learn deeply.

I do not expect you to be able to solve all the issues immediately. Instead, I want to see what you can do on your own. Out in the real world, this is important, since no matter what job you have, you will be expected to seek out information and answers to new topics you have not seen before. This may feel uncomfortable and frustrating. I understand this and want to help you through the process. It helps to remember that there are no mathematical dead-ends! Each time we get stuck, it teaches us something about the problem we are working on, and leads us to a deeper understanding of the mathematics. In the real world though, you are not expected to face your work alone, and in this class is similar.

Asking questions, and explaining things to others, in or out of class, is one of the best ways to improve your understanding of the material. I am always happy to help and will try to give you hints and direction to help you understand the material. At times though, to encourage the exploration process, I may direct you to rethink a problem and to come back to discuss it with me again afterwards. This occurs when I believe that the struggle to understand is imperative for your deep understanding of the material.

- Academic integrity is a fundamental part of the course, which includes meeting deadlines, regular communication, and giving proper reference where it is due. These are essential to course integrity. Feel free to talk to me or each other if you are stuck, but when writing up work, be sure to give acknowledgment where it is due. Submitting someone else's work as your own (PLAGIARISM) is a serious violation of the University's Academic Integrity Code, which defines: "Plagiarism includes, but is not limited to, borrowing, downloading, cutting and pasting, and paraphrasing without acknowledgement, including from online sources, or allowing an individual's academic work to be submitted as another's work."
- I believe that you have the capability to succeed in this course. If you find that you are spending fewer hours outside of class than the university recommends, you can probably improve your understanding and grade by studying more. If you are (on average) spending more hours than these guidelines suggest, you may be working inefficiently; in that case, you should come see me.
- During class, use of technology is allowed only when it is related to our class. Photos or video or audio recordings may not be taken in class without prior permission. Sharing our course materials, ASULearn items, photos, video or audio recordings may not occur without prior permission. There is no eating or drinking allowed in the classroom but you may step out if you need to hydrate or similar. Many activities are designed to be completed during class and you are responsible for all material and announcements, whether you are present or not. You are also responsible for announcements made on the web pages, so check them often.
- We adhere to University-wide syllabus and policy statements:  
<https://academicaffairs.appstate.edu/resources/syllabi-policy-and-statement-information>  
and University policies like on [https://policy.appstate.edu/Policy\\_Manual](https://policy.appstate.edu/Policy_Manual)
- I want you to be informed about your choices regarding what you tell me about certain types of sensitive information. In situations where students disclose experiencing an act of interpersonal violence to their instructor, faculty are required to report that to the campus Title IX Coordinator, who then reaches out to the student by email offering support services. I care about you and want you to get the resources you need. I'm happy to talk with you if you decide you want that, but please be aware that if instead you'd like to explore options with someone who can keep your information totally confidential, I highly recommend the Counseling Center at 828-262-3180. They offer walk-in hours as well as after-hours coverage:  
<http://counseling.appstate.edu>.
- Appalachian Cares is a place to find updates about matters of student health and safety  
<http://appcares.appstate.edu/>.
- Belk Library offers a wide array of research services including access to their digital media studio (with assistance available), video and audio recording rooms, Research Advisory Program sessions in which you

meet individually to work with a librarian, Library guides, study spaces, and online workshops, like <http://library.appstate.edu/gethelp/rap>

- Disco Student Learning Center offers academic resources that will complement and enhance classroom experiences by helping students become acquainted with their studies and learn how to learn effectively. For example, you can meet with a study skills specialist to discuss your goals and develop strategies and study plans to meet those goals or arrange for tutoring. “D.D.,” or Dauphin Disco, was one of the three people who, in 1899, founded Watauga Academy – known as App State today. The word disco is Latin for learn.
- Prerequisite and Co-requisite Material:

It will be helpful to review to make sure you understand concepts from Calculus With Analytic Geometry III, the prerequisite, and know how to do a computational example as we use them in class. This includes acceleration and velocity, arc length, cross product, curvature, cylindrical and spherical coordinates, derivative of a function of one variable whose range is in  $\mathbb{R}^3$ , i.e.  $(x(t), y(t), z(t))$ , directional derivative, dot product, equation of a line in 3-space, equation of a plane, fundamental theorem of calculus, gradient, magnitude, norm or length of a vector, multivariable chain rule, normal vector, parameterizations of curves and surfaces, partial derivatives of a multivariable function i.e.  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  for  $f(x, y)$ , speed, surface area, tangential and normal components of a vector like acceleration, tangent line, tangent plane, tangent vector, and volume. It would be helpful to pay special attention as these concepts as they come up in Linear Algebra, the co-requisite, and in differential geometry, or review them if you have already taken the class: addition of matrices,  $a_{ij}$  matrix notation, basis of a space, determinant of a matrix, dimension of a space, eigenvalue  $\lambda$  from  $A\vec{x} = \lambda\vec{x}$ , linear combination of vectors, multiplication of matrices, including  $A\vec{v}$  and  $AB$ , inverse of a 2x2 matrix, span of vectors, symmetric matrix  $A^T = A$ , transpose of a matrix  $A^T = (a_{ji})$  for  $A = (a_{ij})$ .

## 1.6 Instructor Bio

I am a full Professor of Mathematics, and I am also an affiliate of Gender, Women’s and Sexuality Studies (GWS) and the Math and Science Education Center (MSEC), investigating the connections between mathematics and society. My PhD in Riemannian geometry is from the University of Pennsylvania. I am married to the bassist Joel Landsberg. In our spare time, we like to travel, hike and conduct genealogy research. In addition to my own personal genealogy, I like to give back to the broader community, and in this context, I am also affiliated with ASU’s center for Judaic, Holocaust and Peace Studies. Some of what I like about mathematics is also what I enjoy about genealogy—the sense of exploration, discovery and aha moments that come with lots of patience and effort.