

# Spacetime and Metric Form Research

Dr. Sarah's Differential Geometry

**Welcoming Environment:** Actively listen to others and encourage everyone to participate! Keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter.

Try to help each other! Discuss and keep track of any questions your group has. Feel free to ask me questions during group work time as well as when I bring us back together.

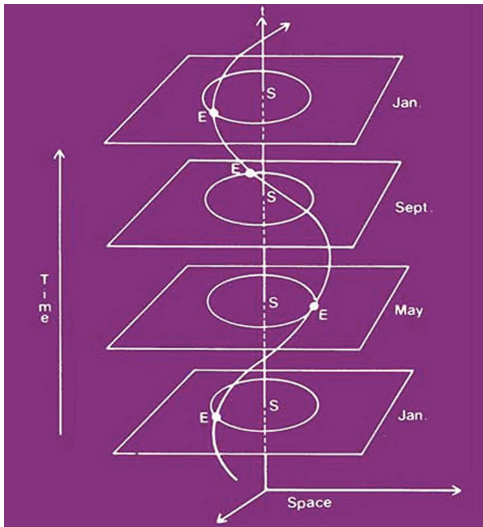
1. If the coordinates of a model for spacetime are  $t, r, \theta, \phi$  then which represents  $ds^2 = g_{ab}dx^a dx^b$ ? Discuss and then respond on [pollev.com/drsarah314](https://pollev.com/drsarah314)

a)  $ds^2 = g_{11}dt^2 - g_{22}dx^2 - g_{33}dy^2 - g_{44}dz^2$

b)  $ds^2 = g_{11}dt^2 + g_{12}dtdr + g_{13}dtd\theta + g_{14}dtd\phi + g_{21}drdt + g_{22}dr^2 + g_{23}drd\theta + g_{24}drd\phi + g_{31}d\theta dt + g_{32}d\theta dr + g_{33}d\theta^2 + g_{34}d\theta d\phi + g_{41}d\phi dt + g_{42}d\phi dr + g_{43}d\phi d\theta + g_{44}d\phi^2$

c) other

2. The cover of *Differential Geometry and Relativity Theory* by Richard Faber, who earned a BS from MIT and PhD in mathematics from Brandeis University, shows



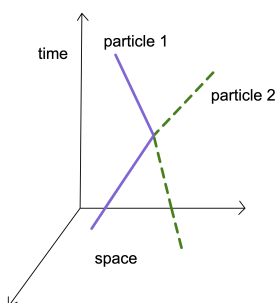
where the earth is rotating around the sun at various points in time. Faber wrote:

As the earth traces out an ellipse in space, time passes. Consequently, the spacetime diagram shows an elliptical coil as the earth's world-line. Slices perpendicular to the  $t$ -axis give the earth's orbital plane at different times of the year. This particular diagram is not drawn to scale.

The diameter of the earth's orbit is about  $3 \times 10^{13} \text{ cm}$ , whereas the length of a year (January to January on the time axis) is about  $9.5 \times 10^{17} \text{ cm}$  [pp. 139–140].

Discuss the visualization with your group as you work to internalize it.

3. What can you say about the 2 particles in this spacetime diagram?



4. Which geodesic is space like? Discuss and then respond on [pollev.com/drsarah314](http://pollev.com/drsarah314)
  - a)  $v^T g_{ij} v = 0$
  - b)  $v^T g_{ij} v > 0$
  - c)  $v^T g_{ij} v < 0$
  
5. If  $I = g_{ab} \dot{x}^a \dot{x}^b$  then a geodesic will satisfy  $\frac{d}{ds} \left( \frac{\partial I}{\partial \dot{x}^a} \right) - \frac{\partial I}{\partial x^a} = 0$ , called the \_\_\_\_\_.  
FYI, the textbook explores this in 7.1 with an eye toward minimization and the calculus of variations rather than solving for Christoffel symbols and curvatures.
  
6. For today, you selected a metric form on ASULearn. If your metric form is named after a person or people, search for their full names. If you have the wormhole metric, search for Flamm, Einstein and Rosen.
  
7. Search for where these people were from and write the country or countries on a board.
  
8. You read “How to Create Your Own Universe in Three Easy Steps” by Lawrence Brenton, who has a BA from the University of Pennsylvania and a PhD in mathematics from the University of Washington. Brenton referred to *Exact Solutions to Einstein’s Field Equations* by Hans Stephani et al., which as of when I last checked, is an ebook we can access from our library via <https://wncln.wncln.org/record=b5525257~S2>. Access the index of the ebook and look for items related to your metric form.
  
9. Our library has a number of ebooks that you can search the index for. Another one is *Relativity in Modern Physics* by Nathalie Deruelle, who has an agrégation from École normale supérieure and PhD from Pierre and Marie Curie University and her prior PhD student Jean-Philippe Uzan. Uzan has a PhD from Paris XI University. Search the index after accessing from the permanent link at (as of when I last checked) <https://wncln.wncln.org/record=b9823281~S2>
  
10. Perform a general library search for your metric form.
  
11. Perform a general web search for your metric form.
  
12. Search for additional information on people related to your metric form including a year or range of years that they may have worked on your metric form.
  
13. The <https://wncln.wncln.org/search~S2/X> search at the library lets us specify Material Type: EBOOKS. Juan Antonio Valiente Kroon, who has a BS from Universidad Nacional Autónoma de México and a PhD in applied mathematics from Queen Mary University of London, is the author of *Conformal Methods in General Relativity*. You can search the library for this or other ebooks.