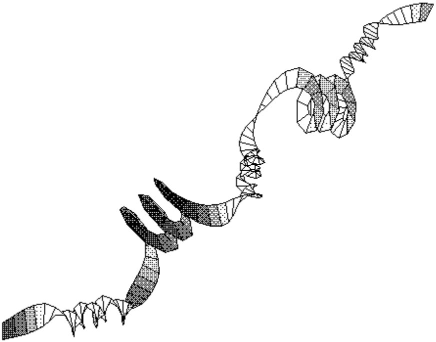
1. For the phone cord curve, which of the following are true?



a) $\kappa = 1$ is constant but $\tau = \sin(s)$ varies

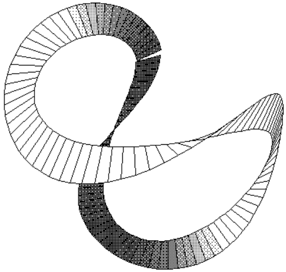
b) $\tau = 1$ is constant but $\kappa = \sin(s)$ varies

c) They are both constant

d) They both vary

e) Rudy Rucker said there is no way to know

2. For the rocker curve, which of the following are true?



a) $\kappa = 1$ is constant but $\tau = \sin(s)$ varies

b) $\tau = 1$ is constant but $\kappa = \sin(s)$ varies

c) This is a baseball stitch curve

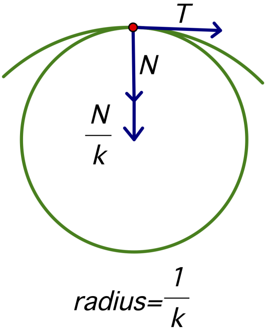
d) More than one of the above

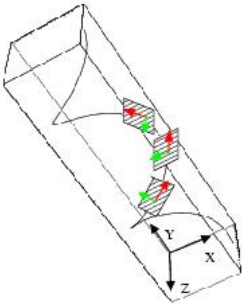
e) None of the above

https://demonstrations.wolfram.com/Intrinsic3DCurves/

Osculating Plane and Osculating Circle

curvature k: tracking T & how the curve *curves*
$-$torsion τ: tracking B & twists out of osculating plane





http://cs-www.cs.yale.edu/homes/li-gang/research/CurveStereo/index.html

osculating circle: radius $\frac{1}{k}$ and center $\alpha(t) \pm \frac{1}{k}N$
osculating plane: $((x, y, z) - \alpha(t)) \cdot B(t) = 0$

Frenet-Serret Frame TNB

- $T = \alpha'(s) = \frac{\alpha'(t)}{|\alpha'(t)|}$. If t is time, then $T = \frac{\vec{v}}{|\vec{v}|} = \frac{\text{velocity}}{\text{speed}}$
- $N = \frac{\vec{\kappa}}{|\vec{\kappa}|} = \frac{\vec{\kappa}}{\kappa}$
  where $\vec{\kappa} = \alpha''(s) = T'(s) = \frac{dT}{ds} = \frac{dT}{dt}\frac{dt}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} = \frac{T'(t)}{|\alpha'(t)|}$
- $B = T \times N$
  $B'(s) = \frac{B'(t)}{|\alpha'(t)|} = -\tau N$
  As your hand moves along a curve, rotate it so the thumb (B) turns away from the middle finger N ($-N$) with a speed of τ. B' captures the movement of the osculating plane $((x, y, z) - \alpha(t)) \cdot B(t) = 0$.

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

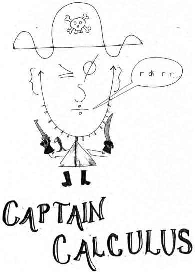
Solve for N, κ, B, τ

Let $\alpha(s) = (\frac{5}{13}\cos(s), -\sin(s), -\frac{12}{13}\cos(s))$

Then $\alpha'(s) = (-\frac{5}{13}\sin(s), -\cos(s), \frac{12}{13}\sin(s))$ and

$|\alpha'(s)| = \sqrt{\frac{25}{169}\sin^2(s) + \cos^2(s) + \frac{144}{169}\sin^2(s)} = 1$

so $T = (-\frac{5}{13}\sin(s), -\cos(s), \frac{12}{13}\sin(s))$



http://www.omniguides.org/3-reasons-why-calculus-is-so-hard/

- Solve for N, κ, B, τ
- What is the shape of this curve?

TNB Derivatives

We defined $T' = \kappa N$
We showed B' had no B component and no T component and thus it makes sense to define $B' = -\tau N$.
T moves towards N and B moves away from N.
Assume these relationships above.

- Show that N' has no N component
- Show that N' has a $-\kappa$ component of T and a τ component of B

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Helix

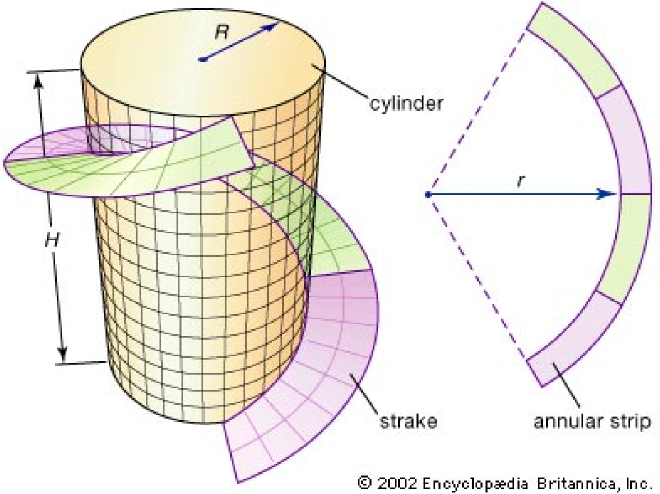
http://www.nerdytshirt.com/calculus3-tshirts.html

Helix

http://www.nerdytshirt.com/calculus3-tshirts.html

Notice that $\dfrac{\tau}{\kappa}$ is constant.

Strake



`https://cdn.britannica.com/22/70822-004-B85BF4BD/`

`strake-strip-dimensions-cylinder-contour-Techniques-differential.jpg`