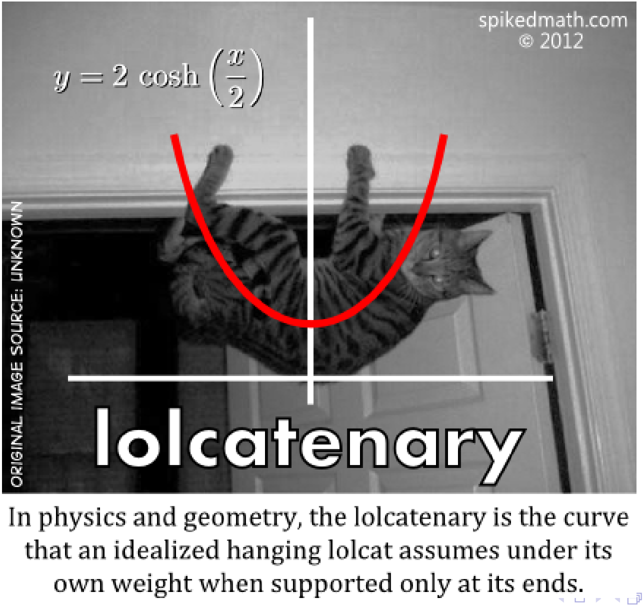
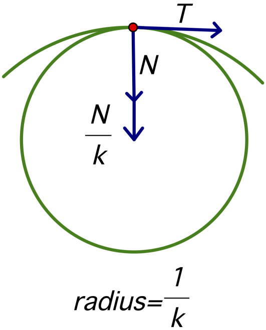
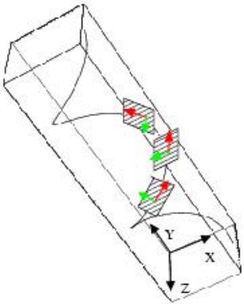
Share from the everyday curve in #1 and/or your favorite non-planar curve in #3.



Osculating Plane and Osculating Circle

curvature k: tracking T & how the curve *curves*
$-$torsion τ: tracking B & twists out of osculating plane





http://cs-www.cs.yale.edu/homes/li-gang/research/CurveStereo/index.html

osculating circle: radius $\frac{1}{k}$ and center $\alpha(t) \pm \frac{1}{k}N$
osculating plane: $((x, y, z) - \alpha(t)) \cdot B(t) = 0$

Frenet-Serret Frame *TNB*

- $T = \alpha'(s) = \frac{\alpha'(t)}{|\alpha'(t)|}$. If t is time, then $T = \frac{\vec{v}}{|\vec{v}|} = \frac{\text{velocity}}{\text{speed}}$
- $N = \frac{\vec{\kappa}}{|\vec{\kappa}|} = \frac{\vec{\kappa}}{\kappa}$

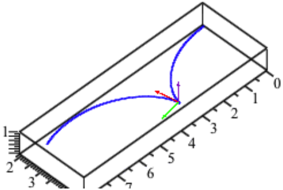
  where $\vec{\kappa} = \alpha''(s) = T'(s) = \frac{dT}{ds} = \frac{dT}{dt}\frac{dt}{ds} = \frac{\frac{dT}{dt}}{\frac{ds}{dt}} = \frac{T'(t)}{|\alpha'(t)|}$
- $B = T \times N$

  $B'(s) = \frac{B'(t)}{|\alpha'(t)|} = -\tau N$

  As your hand moves along a curve, rotate it so the thumb (B) turns away from the middle finger N ($-N$) with a speed of τ. B' captures the movement of the osculating plane $((x, y, z) - \alpha(t)) \cdot B(t) = 0$.

$$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

1. For the Cycloid $\alpha(t) = (t + \sin t, 3 - \cos t, 0)$ is the Frenet Frame defined everywhere on the domain from 0 to 7?



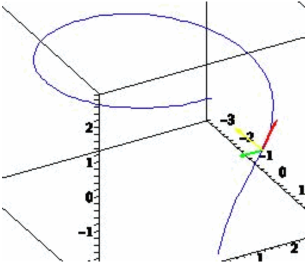
a) Yes and I have a good reason why

b) Yes but I am unsure of why

c) No but I am unsure of why not

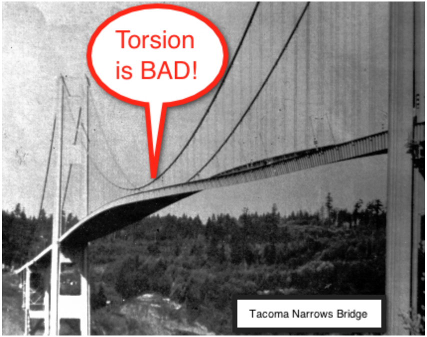
d) No and I have a good reason why not

2. For the Spiral $\alpha(t) = (3\cos t, 3\sin t, \log t)$ is the Frenet Frame defined everywhere on the domain from .0000001 to 2π?



a) Yes and I have a good reason why
b) Yes but I am unsure of why
c) No but I am unsure of why not
d) No and I have a good reason why not

Note: T, k and N work in higher dimensions, but the osculating plane is not defined by a normal, nor does cross product make sense—that is replaced by tensors and forms.



http://pedemmorsels.com/wp-content/uploads/2014/01/Torsion.jpg

- In $\mathbb{R}^3$, show that B' has no B component via a dot product argument.

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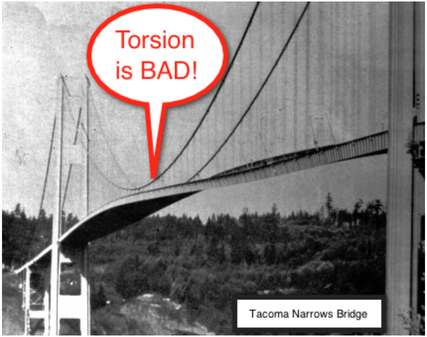

http://pedemmorsels.com/wp-content/uploads/2014/01/Torsion.jpg

- In $\mathbb{R}^3$, show that B' has no B component via a dot product argument.
- Show that B' has no tangential component via a cross product argument.

Warehouse 13's main characters Myka and Pete are trapped in a lemniscate in “The Greatest Gift.” [Syfy, Universal Studios]

`https://drive.google.com/file/d/18mfbulz3AgBwEuNYTo0XZJ8At_BYwp5v/view?usp=sharing`

Warehouse 13's main characters Myka and Pete are trapped in a lemniscate in "The Greatest Gift." [Syfy, Universal Studios]

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a lemniscate can be parameterized so that the metric does expand [Amy Ksir, –]

- Lemniscate of Bernoulli $(\frac{3\cos t}{1+\sin^2 t}, \frac{3\sin t\cos t}{1+\sin^2 t}, 0)$
- Lemniscate of Myka $(\frac{t+t^3}{1+t^4}, \frac{t-t^3}{1+t^4}, 0)$

lemniscatemaple.mw
range=-10 to -.005 and from .005 to 10.