1. Is a latitude (horizontal circle) on a cone a geodesic

a) Yes, for all cone angles, and I have a good reason why
b) Yes, for some but not all cone angles, and I have a good reason why
c) Never and I have a good reason why
d) Yes but I am unsure of why
e) No but I am unsure of why

Recognizing Geodesics on Cone using $\vec{\kappa}_\alpha, \vec{\kappa}_n, \vec{\kappa}_g$

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role of parameters?

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$\vec{x}_u = (\cos v, \sin v, 1), \vec{x}_v = (-u\sin v, u\cos v, 0)$.

$U = \frac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|} = \frac{1}{u\sqrt{2}}(-u\cos v, -u\sin v, u) = \frac{1}{\sqrt{2}}(-\cos v, -\sin v, 1)$

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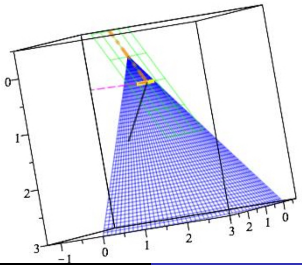
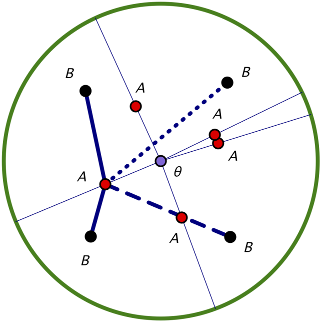
$\vec{\kappa}_\alpha$ (curve's curvature vector): $\frac{T'(t)}{|\alpha'(t)|}$ pink dashed thickness 1

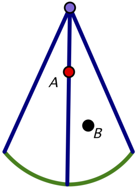
$\vec{\kappa}_n$ (normal curvature): $(U \cdot \vec{\kappa}_\alpha)U$ black solid thickness 2

$\vec{\kappa}_g$ (geodesic curvature): $\vec{\kappa}_\alpha$ - $\vec{\kappa}_n$ tan dashdot style thickness 4



2. How many different geodesics are there between A and B on this cone that has an angle a bit less than $\frac{\pi}{2}$?





a) less than 4

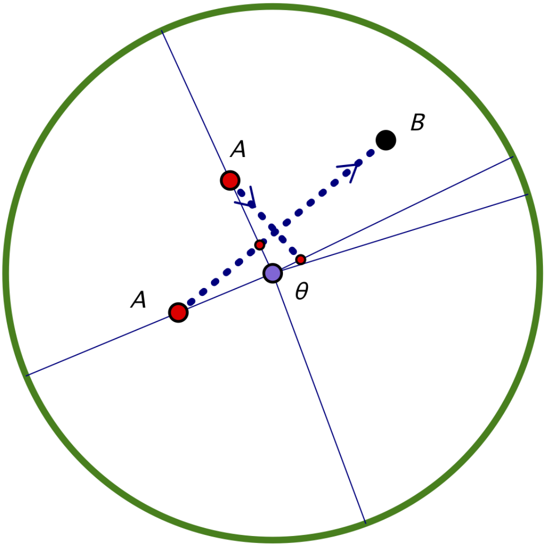
b) 4

c) more than 4

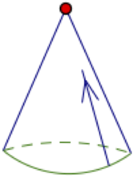
Next, what are the shapes of the geodesics between A and B on this cone? Sketch each on a cone and include in your sketch

—front or back of the cone (or both)

—any intersections of a geodesic with itself


B
A
A
θ

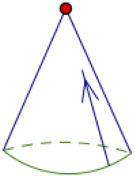
3.



What happens when a bug gets to the cone point along this vertical geodesic?

a) The geodesic ends there.
b) The bug can continue to walk straight through the cone point to the "other side" by bisecting the cone angle there.
c) other

3.



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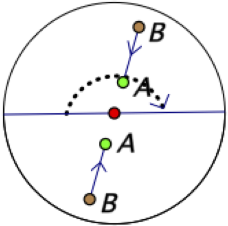
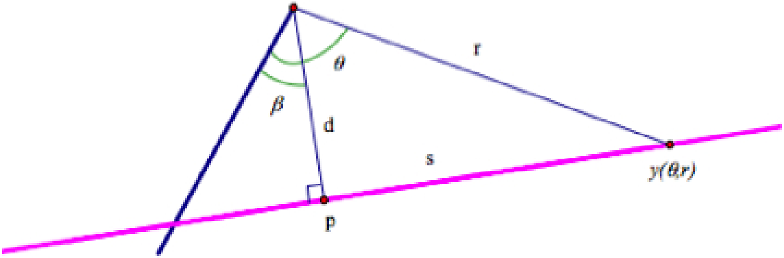


4. Which is an equation of a geodesic that an arbitrary point $y(\theta, r)$ satisfies, where d and β are defined as in the hw and following picture:



a) $r = d\sec(\theta - \beta)$

b) $d = r\sec(\theta - \beta)$

c) both

d) other

5. In general on a cone of small enough cone angle, a geodesic

a) won't intersect itself
b) will intersect itself a finite number of times with a maximum crossing number that depends on the specific cone angle
c) will intersect itself infinitely many times

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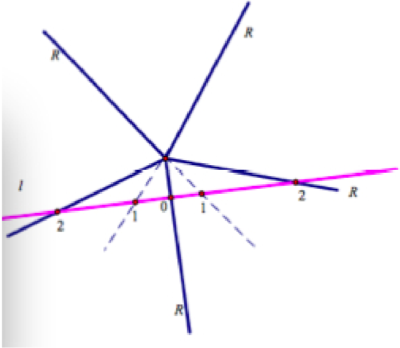
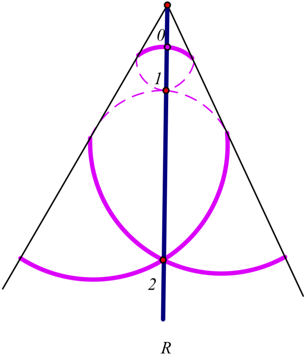
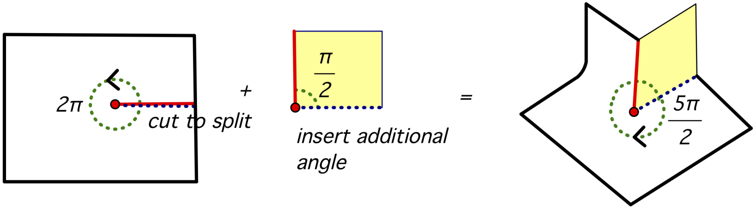




6. Extend the $\frac{5\pi}{2}$ cone in all directions so that it continues indefinitely. Can we find a point P (other than the cone point) and a geodesic l (not through the cone point) such that there is more than 1 geodesic through P that does not intersect l?



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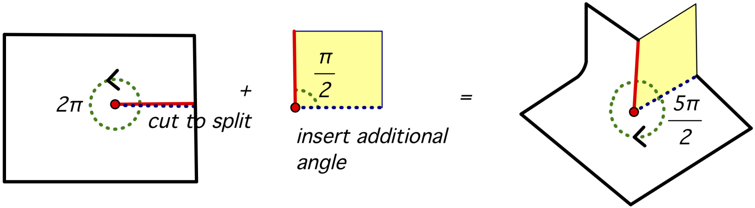

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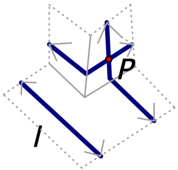
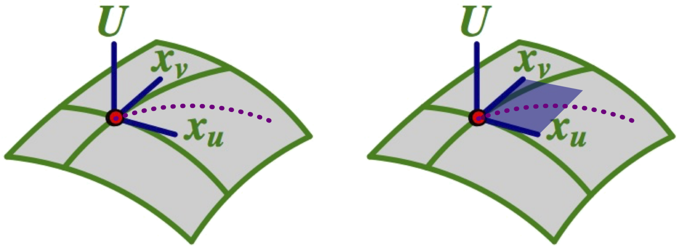


First Fundamental Form and Plane



- Surface $\mathbf{x}(u, v)$ and curve $\alpha(t)$ on it given by $u(t)$ & $v(t)$.
  $\alpha'(t) =$

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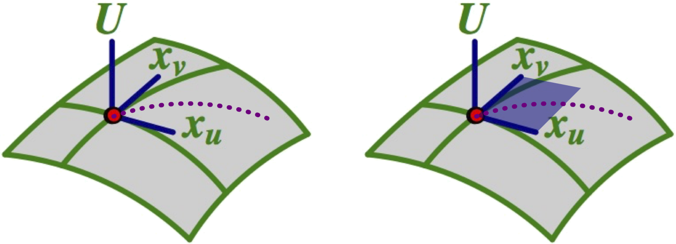

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  $\alpha'(t) = \vec{x}_u \frac{du}{dt} + \vec{x}_v \frac{dv}{dt}$

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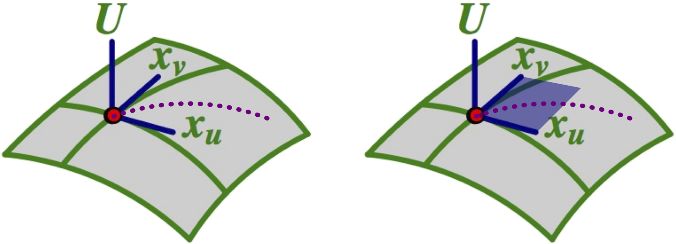

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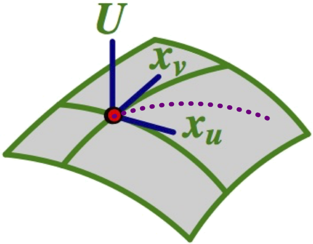

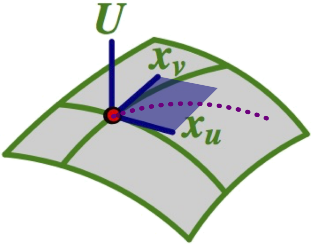

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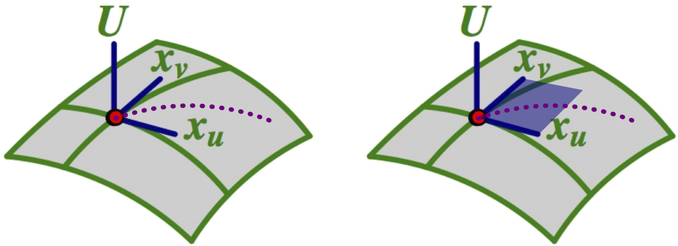

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  $= \vec{x_u}\cdot\vec{x_u}(\frac{du}{dt})^2 + 2\vec{x_u}\cdot\vec{x_v}\frac{du}{dt}\frac{dv}{dt} + \vec{x_v}\cdot\vec{x_v}(\frac{dv}{dt})^2$

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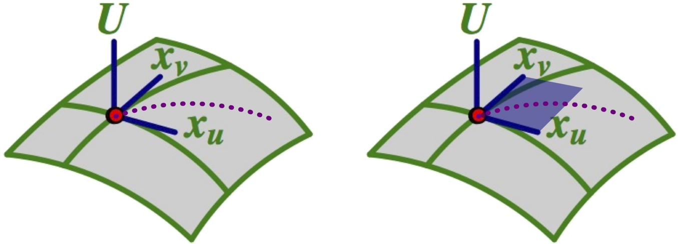

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  $= E(\frac{du}{dt})^2 + 2F\frac{du}{dt}\frac{dv}{dt} + G(\frac{dv}{dt})^2$
  $ds^2 = g_{11}(du^1)^2 + 2g_{12}du^1du^2 + g_{22}(du^2)^2 = \sum_{i,j} g_{ij}du^idu^j$
- $\mathbf{x}(u, v) = (u, v, 0)$ compared to $\mathbf{x}(u, v) = (u\cos v, u\sin v, u)$