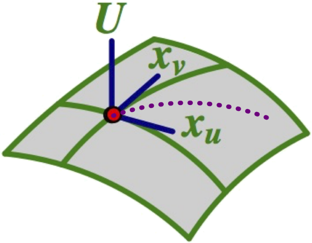
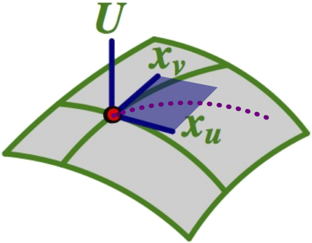
# First Fundamental Form: Plane and Cone




- Regular surface $M = \mathbf{x}(u, v)$, where $\vec{x}_u \times \vec{x}_v \neq 0$, and $u(t)$ & $v(t)$ give curve $\alpha(t)$. Then $\vec{x}_u, \vec{x}_v$ form basis for $T_pM$ and $\alpha'(t) =$

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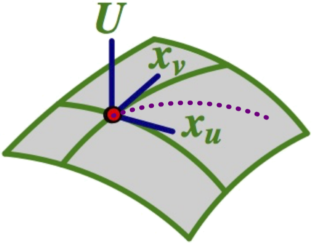


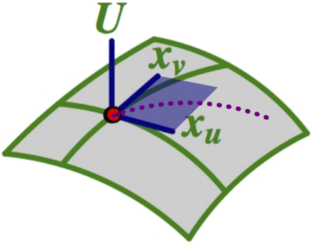


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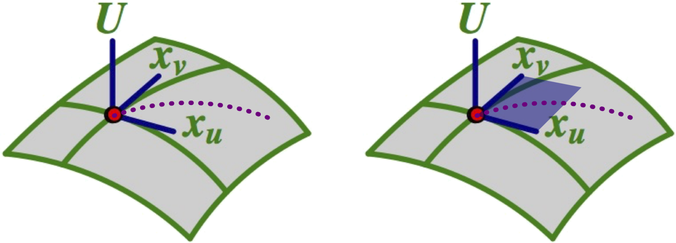


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  $\alpha'(t) = \vec{x_u}\frac{du}{dt} + \vec{x_v}\frac{dv}{dt}$
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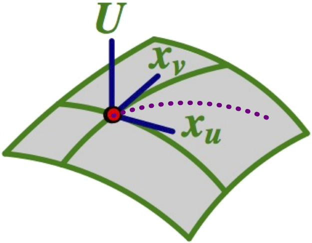


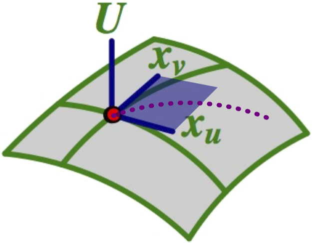


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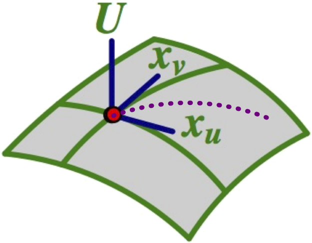


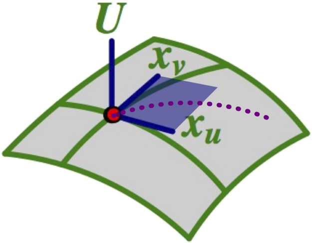


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  $= \vec{x_u} \cdot \vec{x_u}(\frac{du}{dt})^2 + 2\vec{x_u} \cdot \vec{x_v}\frac{du}{dt}\frac{dv}{dt} + \vec{x_v} \cdot \vec{x_v}(\frac{dv}{dt})^2$

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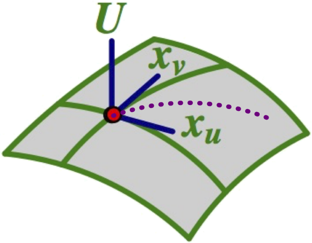


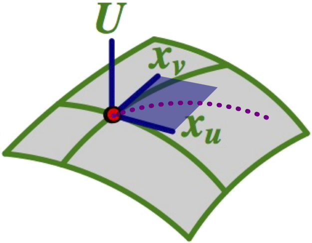


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  $\alpha'(t) = \vec{x}_u \frac{du}{dt} + \vec{x}_v \frac{dv}{dt}$
  $(\frac{ds}{dt})^2 = |\alpha'(t)|^2 = \alpha'(t) \cdot \alpha'(t) = (\vec{x}_u \frac{du}{dt} + \vec{x}_v \frac{dv}{dt}) \cdot (\vec{x}_u \frac{du}{dt} + \vec{x}_v \frac{dv}{dt})$
  $= \vec{x}_u \cdot \vec{x}_u (\frac{du}{dt})^2 + 2\vec{x}_u \cdot \vec{x}_v \frac{du}{dt}\frac{dv}{dt} + \vec{x}_v \cdot \vec{x}_v (\frac{dv}{dt})^2$
  $= E(\frac{du}{dt})^2 + 2F\frac{du}{dt}\frac{dv}{dt} + G(\frac{dv}{dt})^2$
  $ds^2 = g_{11}(du^1)^2 + 2g_{12}du^1du^2 + g_{22}(du^2)^2 = \sum_{i,j} g_{ij}du^idu^j$
- $\mathbf{x}(u, v) = (u, v, 0)$ compared to $\mathbf{x}(u, v) = (u\cos v, u\sin v, u)$

## First Fundamental Form $E = \vec{x_u} \cdot \vec{x_u}, F = \vec{x_u} \cdot \vec{x_v}, G = \vec{x_v} \cdot \vec{x_v}$

- Matrix representation: $g_{ij} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} E & F \\ F & G \end{bmatrix}$
- $g_{ij}$ determines dot products of tangent vectors $\vec{w}_1$, $\vec{w}_2$ in $T_pM$

$\{\vec{x_u}, \vec{x_v}\}$ is a basis: $\vec{w}_1 = a\vec{x_u} + b\vec{x_v}$ , $\vec{w}_2 = c\vec{x_u} + d\vec{x_v}$

$\vec{w}_1 \cdot \vec{w}_2 \overset{\text{foil}}{=}$

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$$\vec{w}_1 \cdot \vec{w}_2 \overset{\text{foil}}{=} ac\vec{x}_u \cdot \vec{x}_u + (ad + bc)\vec{x_u} \cdot \vec{x}_v + bd\vec{x}_v \cdot \vec{x}_v$$
$$= acE + (ad + bc)F + bdG$$

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$$\begin{aligned}
\vec{w}_1 \cdot \vec{w}_2 &\overset{\text{foil}}{=} ac\vec{x}_u \cdot \vec{x}_u + (ad + bc)\vec{x_u} \cdot \vec{x}_v + bd\vec{x}_v \cdot \vec{x}_v \\
&= acE + (ad + bc)F + bdG = a(cE + dF) + b(cF + dG) \\
&= \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} cE + dF \\ cF + dG \end{bmatrix} \\
&= \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}
\end{aligned}$$

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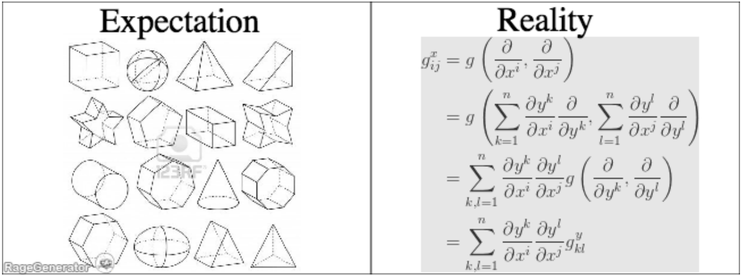
$$= \begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$
$$= \vec{w}_1 \cdot \vec{w}_2 = |\vec{w}_1||\vec{w}_2| \cos\theta$$

E, F, G play important roles in many intrinsic properties of a surface like length $(\frac{ds}{dt})^2$, area (det) and angles (above)

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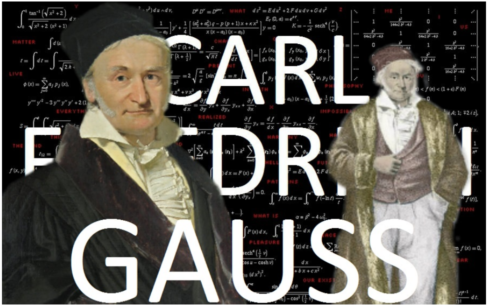
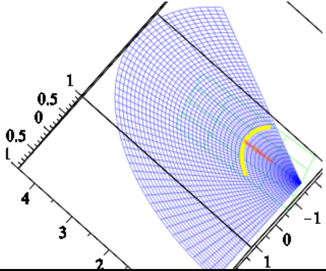
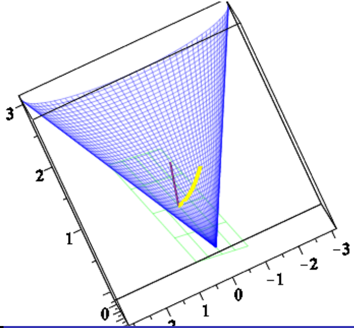



http://vignette1.wikia.nocookie.net/epicrapbattlesofhistory/images/8/83/Pizap.com14239077419801.jpg

# Maple file coneandplaneforms.mw

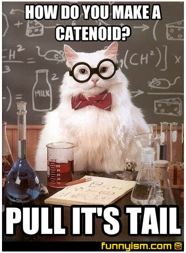
- new plane $[\sqrt{2}x\cos(\frac{y}{\sqrt{2}}), \sqrt{2}x\sin(\frac{y}{\sqrt{2}}), 0]$ that is isometric to the cone $[x\cos(y), x\sin(y), x]$
- longitude and latitude on the new plane
- first fundamental form of the new plane and cone
- using secant to write the geodesic between the points $(1, 0, 1)$ and $(0, 1, 1)$ on the cone (i.e. the point $x = 1$ and $y = 0$ and the point $x = 1, y = \frac{\pi}{2}$ (see p. 247–248 for more information)

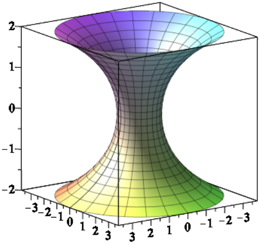


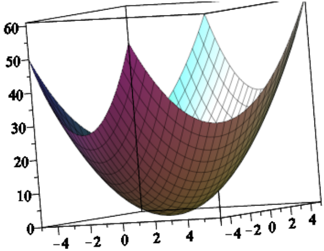



For homework today you were to read section 2.1. Write down examples of surfaces for each type of parametrization.

- surface of revolution $x(u,v) = (g(u), h(u)\cos v, h(u)\sin v)$ from a planar curve $\alpha(u) = (g(u), h(u), 0)$
- ruled surface $x(u,v) = \beta(u) + v\delta(u)$, where $\beta$ and $\delta$ are curves and $x(u,v)$ is lines emanating from the directrix $\beta$ going in the direction of $\delta$
- Monge patch $x(u,v) = (u, v, f(u,v))$
- geographical coordinates $x(u,v) = (R\cos u\cos v, R\sin u\cos v, R\sin v)$




http://www.funnyism.com/i/memefactory/how-do-you-make-a-catenoid-pull-its-tail