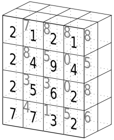
Tensors and the Metric Tensor g_{ij}

't'
'e'
'n'
's'
'o'
'r'

3	1	4	1
5	9	2	6
5	3	5	8
9	7	9	3
2	3	8	4
6	2	6	4



lists #s   vectors   stack of matrices

- algebraic combinations of vectors, matrices, vector spaces, algebras, modules or other structures
- often geometrically meaningful
- not all tensors are inherently linear maps

g_{ij} inner products of tangent vectors $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$

$$g^{ij} = g_{ij}^{-1} = \begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1} = \frac{1}{EG-F^2} \begin{bmatrix} G & -F \\ -F & E \end{bmatrix}$$

SpaceTime-Time

Surfaces: g_{ij} inner products of tangent vectors $w^T g_{ij} v$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

SpaceTime: Metric tensor is now 4x4 symmetric matrix acting as $w^T g_{ij} v$ on (t, x, y, z) vectors.

$$\begin{bmatrix} t & x & y & z \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix}$$

Yardstick plus clock!

Christoffel Symbols and Curvatures

- Write $I = g_{ab}\dot{x}^a\dot{x}^b$.
- Let $a = 1$ in the Euler-Lagrange Equation $\frac{d}{ds}(\frac{\partial I}{\partial \dot{x}^a}) - \frac{\partial I}{\partial x^a} = 0$ for all a.
- Take the relevant partials and derivatives.
- Write the expansion of the geodesic equation $\ddot{x}^a + \Gamma^a_{bc}\dot{x}^b\dot{x}^c = 0$ using $a = 1$ (b, c range from 1...4)
- Compare to find the Γ^1_{ab} Christoffel symbols?
- Repeat for $a = 2$, $a = 3$, $a = 4$

Riemann curvature tensor or Riemann-Christoffel tensor
$R^a_{bcd} = \partial_c\Gamma^a_{bd} - \partial_d\Gamma^a_{bc} + \Gamma^e_{bd}\Gamma^a_{ec} - \Gamma^e_{bc}\Gamma^a_{ed}$
Ricci tensor $R_{ab} = R^c_{acb} = g^{cd}R_{dacb}$
Scalar curvature $R = g^{ab}R_{ab}$
Einstein tensor $G_{ab} = R_{ab} - \frac{1}{2}g_{ab}R$

Christoffel Symbols and Curvatures

The Christoffel symbols

- intrinsic quantities, how to take covariant derivatives
- coefficients of tangent vectors (connection coefficients)
- measure whether or not vectors are parallel transports
- in relativity, gravitational forces. geodesics and curvatures.

Riemann curvature tensor: measures how much a manifold is not flat via $4^4 = 256$ entries for spacetime.

Ricci tensor: trace (sum of diagonal elements) relates to the metric volume $\sqrt{det g_{ij}}$.

Scalar curvature: number. For surfaces—twice Gaussian curvature. For relativity—Lagrangian density.

Einstein tensor describes curvature of spacetime due to the presence of energy or mass, has zero divergence


TENSOR CALCULUS
A quick and fun way to learn tensor calculus for ages 6 and up.
E=mc 2
James Clerk Maxwell's Silver Hammer!
9
Field Equations the easy way!
3
$R_{ab} - \frac{1}{2} R g_{ab} = \frac{8\pi G}{c^4} T_{ab}.$
Hey kids! The professor has a fun game to play! you will learn this with the geodesic jig! You can sing and dance the energy momentum tensor and tell your friends space-time can be non - Euclidian. Once you are done, draw 3!
Christoffel Symbols can be fun!
draw 2
These guys play a trick on you! Look closely kids! Are they tensors? No! Why? let the song teach you! They go hand in hand with the geodesic. Time to do the time warp! Once you have learned, draw 2!
8
Black holes rotate. So can you!