## Surfaces

Open your eyes to surfaces in our world. Find, either in person or in a picture, a surface which interests you and be prepared to share where it arises and what significance it has

How did you select your surface?

- a) aesthetics
- b) applicability
- c) interesting name
- d) more than one of the above
- e) other

Share the significance and surface with your neighbors.

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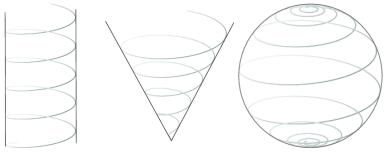
- a) aesthetics
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sphere versus ball

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#### Helix, Loxodrome or Rhumb Line



http://www.tankonyvtar.hu/en/tartalom/tamop425/0038\_matematika\_Miklos\_

Hoffmann-Topology\_and\_differential\_geometry/images/csavar\_hengeres\_kupos\_cdr.png

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### **Isoperimetric Inequality**



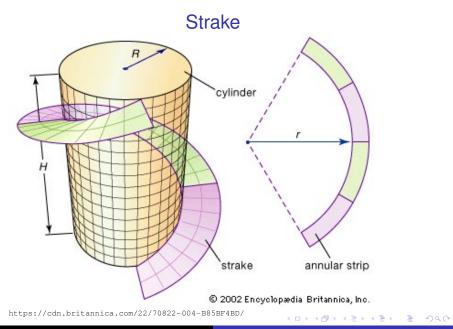
"Dear Sir: Why do you sell dog food in tall cans and sell cat food in short cans?"



"Neither of these shapes is the optimal use of surface area when compared to volume."

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#### Surfaces

Each point has a neighborhood that locally looks like a plane.

#### manifolds and surfaces versus orbifolds



Google Earth

Google Earth

Dr. Sarah Math 4140/5530: Differential Geometry

### Visual/Tactile Perspectives

At a very early age, children develop a very rich 'visual intelligence' in terms of perception and experiences. They have questions and lots of these questions and explorations can be connected to geometry if we use the right types of physical and visual presentations... the precise vocabulary is... differential geometry and differential topology [28 (Hoffmann, 1998), 36 (Koenderink, 1990)]... we should connect with these abilities. (Whiteley, 1999)

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## **Intrinsic Perspectives**

- What is the dimension of a surface?
- What is straight on a surface?



## SURFACE TENSION

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# Geodesic—Intrinsically Straight Path

• Dictionary.com: locally minimizes the distance. Equivalently, it is a path of minimal curvature.

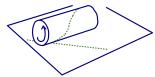


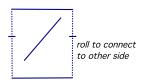
# Geodesic—Intrinsically Straight Path

- Yan-Bin Jia: The term geodesic comes from the science of geodesy, which is concerned with measurements of the earth's surface...
  - Carl Gustav Jacobi (1804–1851) studied the "shortest curves" on an ellipsoid of rotation which he referred to as "geodesic curves"



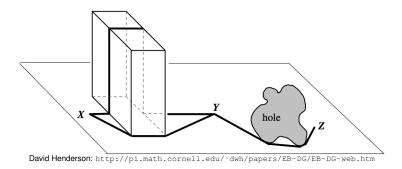
## (Intrinsically Straight) Geodesics on a Cylinder



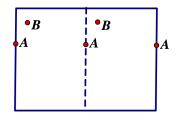




- symmetry and our feet
- rolling arguments (covering arguments in general): draw a straight line on a piece of paper, roll up and connect the edges to form a cylinder and see what the line becomes
- 1.Can a geodesic ever intersect itself? Why?
- 2. How many differently shaped geodesics can you find? What do they look like? Explain.
- 3. Is straight always shortest distance? Explain.
- 4. Is shortest distance always straight? Explain.
- 5. How many geodesics join 2 points? Why?



If a surface is smooth (in the  $C^2$  sense, with local coordinates whose first and second derivatives exist and are continuous), then a geodesic on the surface is always the locally shortest path between "nearby" points. If the surface is also geodesically complete (that is, every geodesic on it can be extended indefinitely, for example, there are no holes), then any two points can be joined by a geodesic that is the shortest path. (Intrinsically Straight) Geodesics on a Cylinder 5. How many geodesics join 2 points? A 2-sheeted covering:



- Fold a paper in half vertically so you have 2 equal regions
- Label point A on each edge at the same height (3 As)
- Choose Bs not on the same vertical or horizontal line as A
- Draw a line between every A and every B. Marker is best.
- Roll the sheet up so As match & examine the geodesics