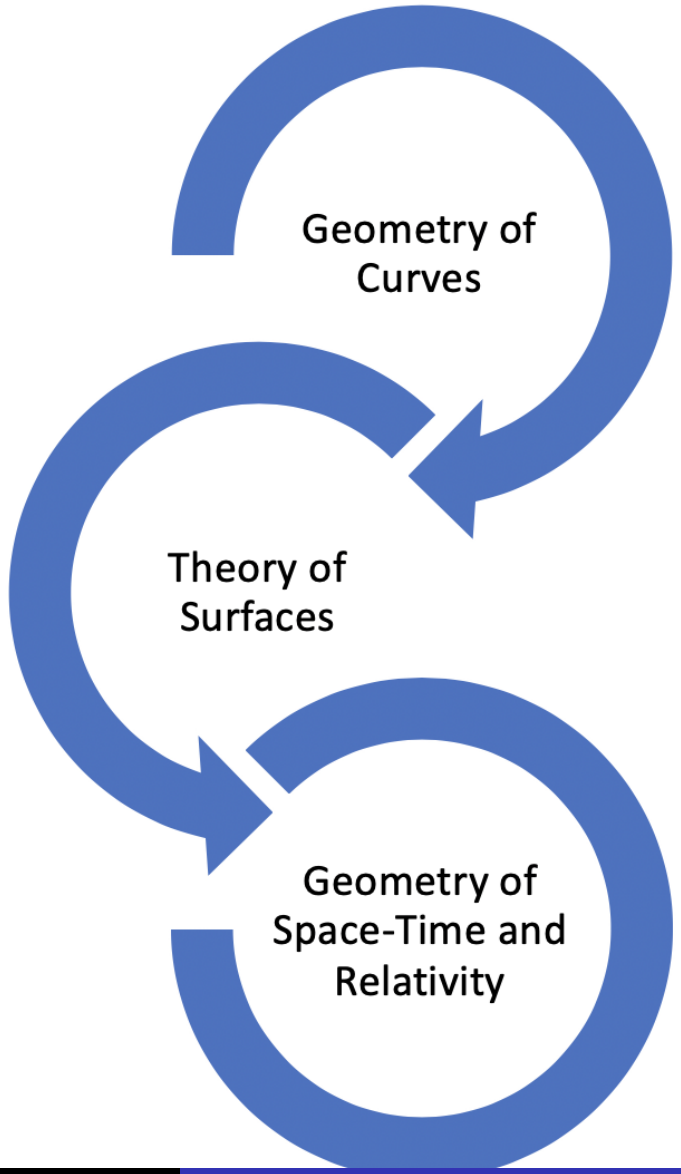
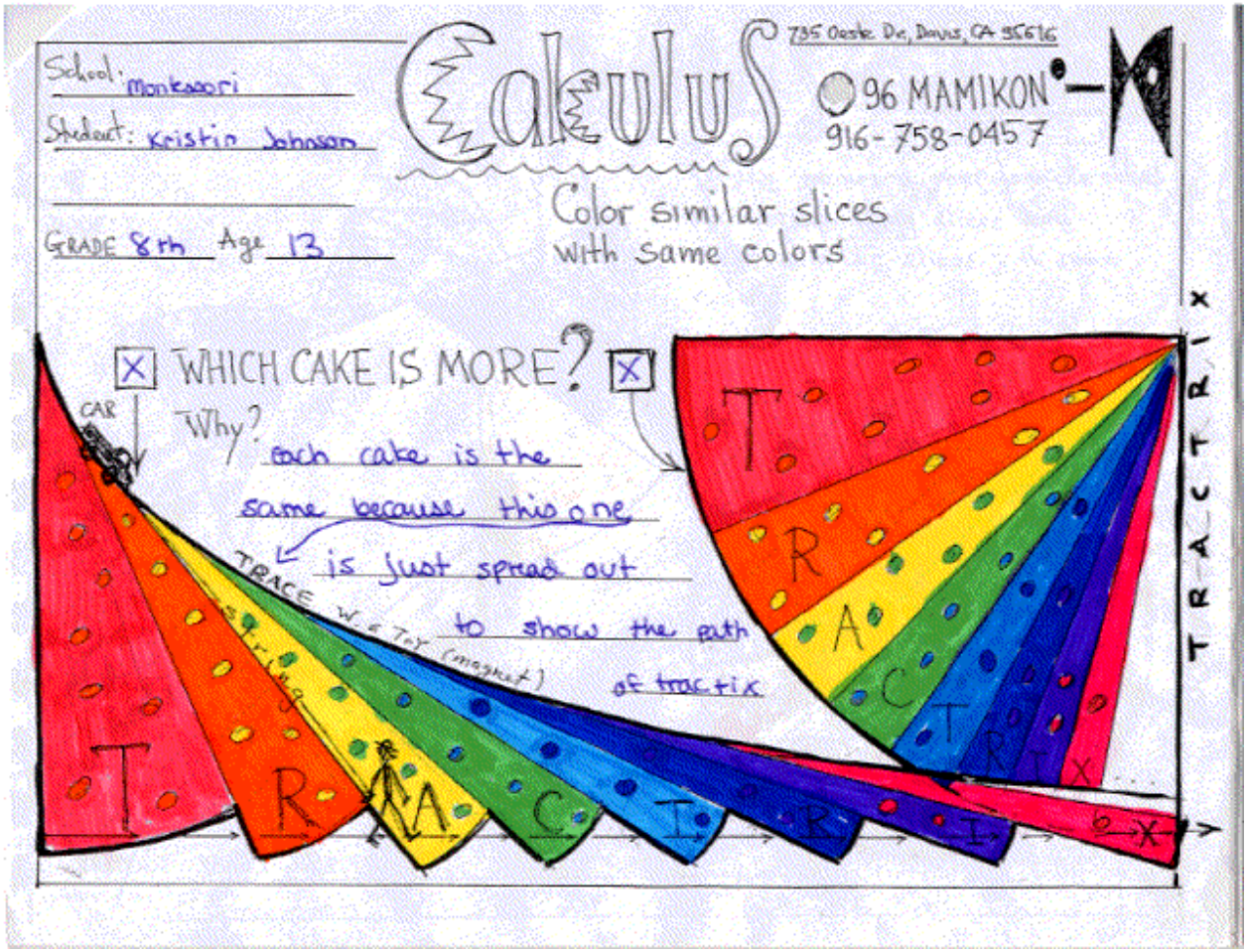
- Turn in homework 1 up front
- Share what problems you didn't turn in with your neighbors and discuss the meaning/geometry/physics of:
  1. $\vec{v} - \vec{w}$
  2. $\vec{v} + \vec{w}$
  3. $\vec{v} \cdot \vec{w}$
  4. $\vec{v} \times \vec{w}$
  5. $\arccos(\frac{\vec{v}\cdot\vec{w}}{|v||w|})$
  6. $(\frac{\vec{v}\cdot\vec{w}}{|w||w|})\vec{w}$



# 1.1 & 1.2: Arc Length

$$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\tfrac{\theta}{2})), \sin(\theta)) \quad \theta \in (\tfrac{\pi}{2}, \tfrac{2\pi}{3})$$

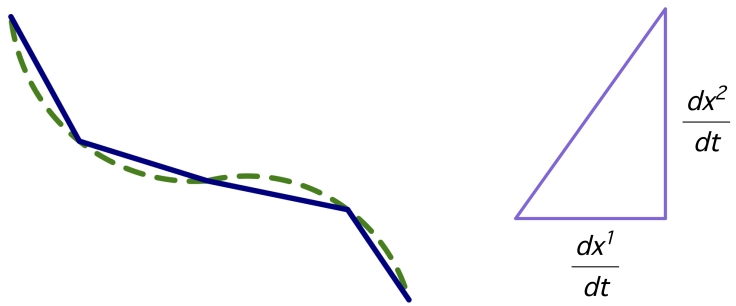



https://www.its.caltech.edu/~mamikon/Mont.html

# Importance of Arc Length $s$

Let arc length $s(t) = \int_a^t |\alpha'(u)| du$

- measures the length of a curve by adding up infinitesimal linear approximation (Pythagorean theorem metric)
- simplifies computations




$$|\alpha'(t)| = \sqrt{(\frac{dx^1}{dt})^2 + (\frac{dx^2}{dt})^2 + \cdots}$$

Prove: If $\alpha$ is a differentiable curve that is regular then $\alpha$ can be reparameterized by arc length $s$ to have unit speed/tangent (even though we might not write down a closed form solution explicitly)

# Frenet Frame: T in TNB

The unit tangent in the direction of motion is given a special name in differential geometry and its applications: $T$

$T(s) = \alpha'(s)$

$T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$ $\frac{\textit{velocity}}{\textit{speed}}$

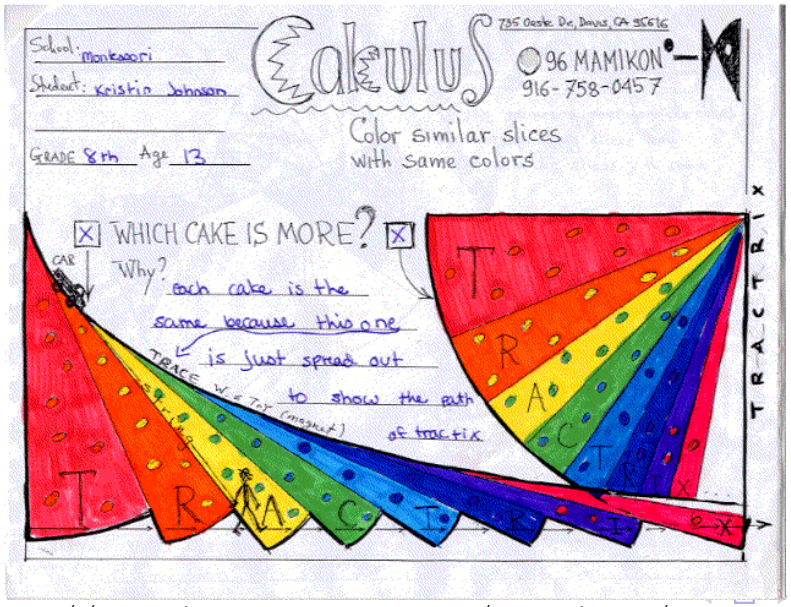

http://www.nerdytshirt.com/images/shirt-images/calculus-3/

pythagorean-magnitude-t-shirt-24.jpg

# Arc length and $T$ of Tractrix

$$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\tfrac{\theta}{2})), \sin(\theta))$$

- Compute arc length $s(t) = \int_{\frac{\pi}{2}}^{t} |\alpha'(\theta)| d\theta$
- Write the inverse function $t(s)$ by solving for $t$
- Reparameterize the curve by arc length $\beta(s) = \alpha(t(s))$
- Compute unit tangent $T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$
- Visualize in Maple

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# Arc Length and $T$ of Helix

Work with neighbors or check-in with them regularly:

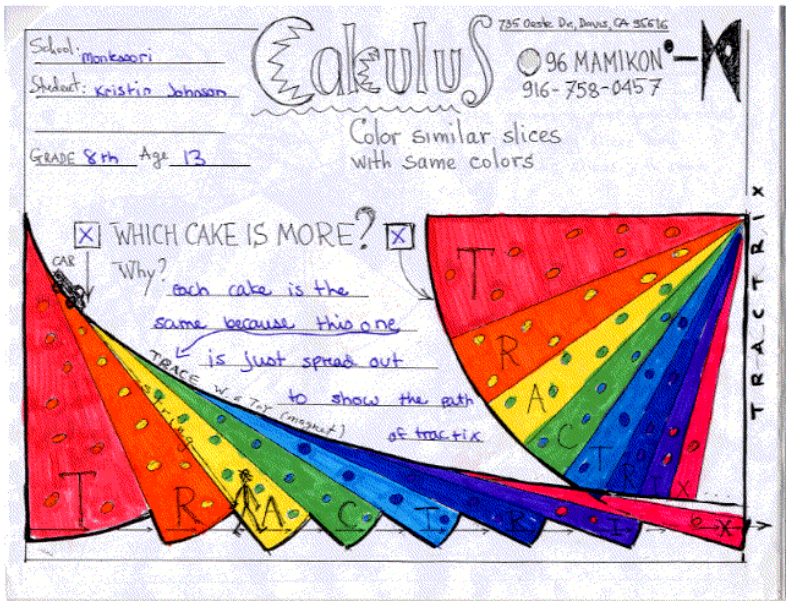
$\alpha(t) = (a\cos(t), a\sin(t), bt)$ where $a, b \in \mathbb{R}$ constants

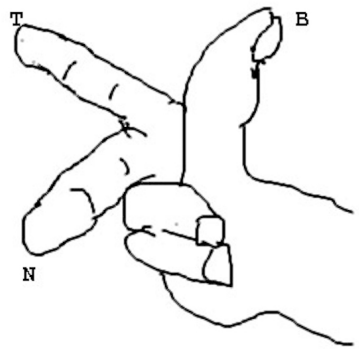
- Compute unit tangent $T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$
- Compute arc length $s(t) = \int_0^t |\alpha'(u)|du$
- Write the inverse function $t(s)$ by solving for $t$
- Reparameterize the curve by arc length $\beta(s) = \alpha(t(s))$

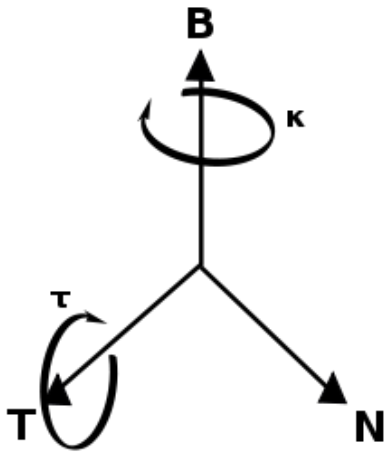
http://previews.123rf.com/images/limbi007/limbi0071302/limbi007130200034/17726502-Orange-cartoon-characters-runs-on-the-green-helix-Stock-Photo-orange-spiral.

# Differential Geometry of Tractrix

$$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\tfrac{\theta}{2})), \sin(\theta))$$

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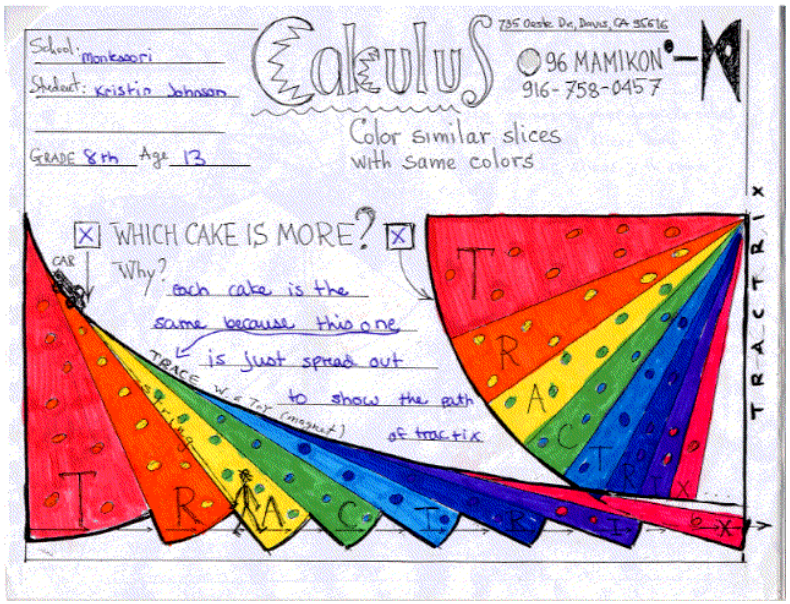
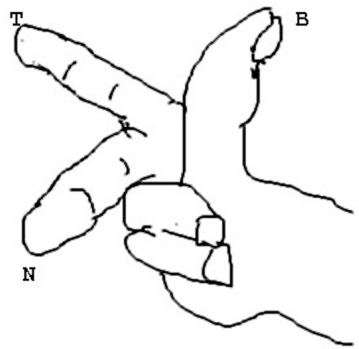
http://www.rudyrucker.com/transrealbooks/collectedessays/images/kaptauhand.jpg

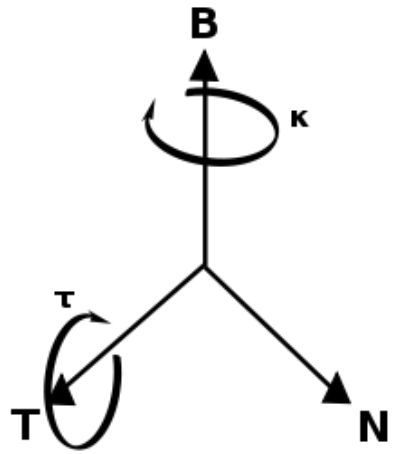
- velocity, acceleration, jerk, and higher time derivatives

# Differential Geometry of Tractrix

$$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\tfrac{\theta}{2})), \sin(\theta))$$

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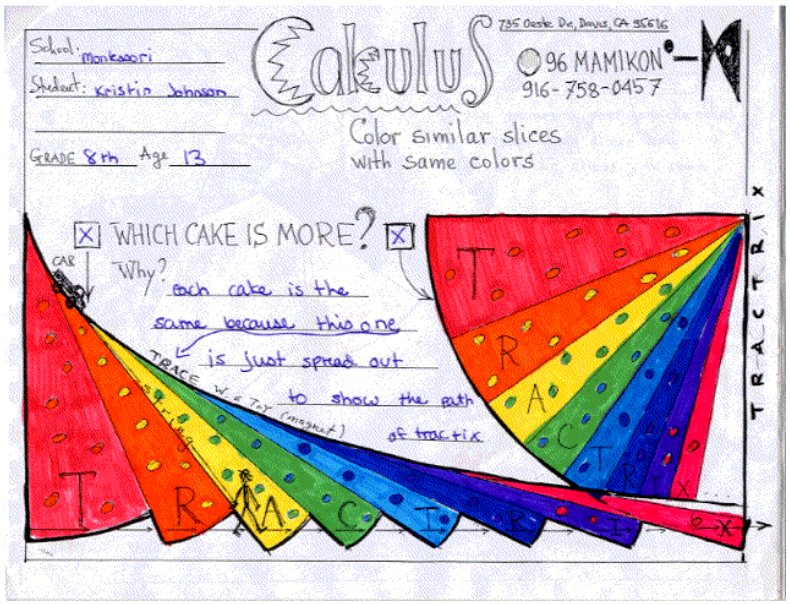
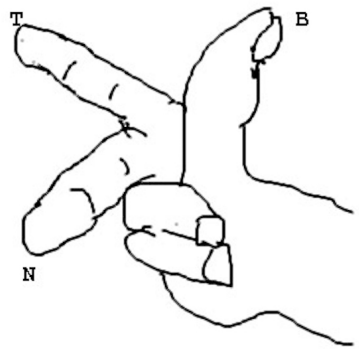
http://www.rudyrucker.com/transrealbooks/collectedessays/images/kaptauhand.jpg

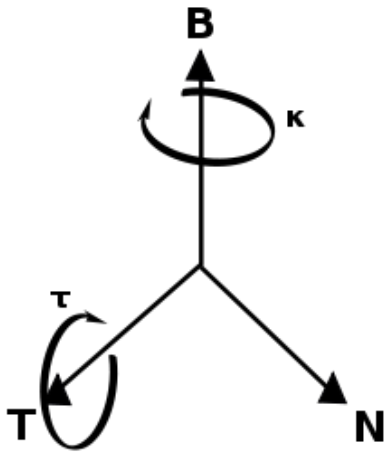
- velocity, acceleration, jerk, and higher time derivatives
- speed and arclength

# Differential Geometry of Tractrix

$$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\tfrac{\theta}{2})), \sin(\theta))$$

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- velocity, acceleration, jerk, and higher time derivatives
- speed and arclength
- TNB Frame, curvature and torsion