Additional Graduate Problems for Project 2

Graduate students will complete the undergraduate assignment and, in addition, turn in your responses to the following, so you will have 2 additional problems turned in, with some choice for the first one. See the instructions in the undergraduate assignment, which apply here too.

1. Select one of the following:

   (a) **Graduate Problem: Geodesic Completeness**
   Read 5.3 p. 225-226 on geodesic completeness and then respond to Exercise 5.3.3

   OR

   (b) **Graduate Problem: Eigenvalues of the Shape Operator**
   Explore the eigenvalues of the shape operator on your surface at one or more revealing points. I already have a procedure written, so if you add commands like

   ```
   eigenvaluesshape(X);
   evalf(subs(u=0,v=0,eigenvaluesshape(X)[1]));
   evalf(subs(u=0,v=0,eigenvaluesshape(X)[2]));
   ```

   you can explore that way, as one possibility. You can use the stop sign on the computation is Maple if taking too long on the first command, but I did test it out on most of the surfaces where it ran fine, even if lengthy as a computation. Connect the Maple output to intuition about the surface in terms of the principal curvatures. Explain what point(s) you selected and what happens there.

   OR

   (c) **Graduate Problem: Surface Area**
   Integrate your finite surface area region analytically and/or numerically and show reasoning or Maple work.

2. **Graduate Problem: C^1 isometric embedding of a flat torus in $\mathbb{R}^3$**

   There is a C^1 isometric embedding of a flat torus in $\mathbb{R}^3$. Read pages 7–9 (Isometric embeddings: from Schlaefli to Nash, but stop at The Gromov Convex Integration Theory) of Borrelli, Vincent, Säid Jabrane, Francis Lazarus, and Boris Thibert (2013). "Isometric embeddings of the square flat torus in ambient space." *Ensaios Matemáticos* 24 pp. 1–91, which is available at `https://www.emis.de/journals/em/images/pdf/em_24.pdf`. Summarize what Borelli et al. say about why C^1 is possible while C^2 is impossible, and what they say about curvatures and the Gauss map.