Project 1: Research, Investigate, and Present a Curve

You may work alone or with one other person and turn in one per group to one of your ASULearns. Curves are on a **first come-first-served** basis in the ASULearn choice selection feature.

Explore the following questions via the sources and Maple file I provided for you as well as researching and analyzing yourself. (**Keep track of ALL references for # 15**). **Write it up in your own words in the language of our class** but you may use pictures from elsewhere (with proper reference).

You will turn in all of the following and share with your classmates (see #17).

1. List your preferred first name(s). If you are turning this in with a partner, list both names.
2. Search an image database for "differential geometry of *", where * is a name of your curve that you selected in the choice feature on ASULearn. You might use Google images, for instance. Provide one or more interesting images that relate. Be sure to list any picture references (and any other references) in #15. Google is a database, not typically the original source of an image, so be sure to track back to the original source.
3. Handwrite or professionally typeset general formulas for the following entities as a review in equations and/or words. Assume that you have a curve parametrized in time. Do NOT do any calculations for your specific curve here, but do show generic formulas connected to the language of our class and/or explain how to calculate each from a parameterization of the curve $\alpha(t)$ itself. Your answers may build upon one another, i.e. using part (a) in another part.

   (a) velocity (generally—NOT for your specific curve)
   (b) acceleration
   (c) jerk
   (d) speed
   (e) arc length s
   (f) curvature vector $\vec{\kappa}$
   (g) curvature κ
   (h) torsion τ
   (i) T
   (j) N
   (k) B

   For instance, for velocity, you might write something like: "The velocity is the first derivative of position componentwise with respect to time so if $\alpha(t) = (x(t), y(t), z(t))$ then the velocity is $\alpha'(t) = (x'(t), y'(t), z'(t))$"
4. Explain in your own words what each of the items from the last question generally means physically and/or geometrically, connected to the language of our class. For instance, "Velocity gives us a tangent to the curve at the given point. The velocity represents the way the position is changing in time as both a direction and a number, the speed, which is the length of the velocity vector. A tangent vector is the best fit line to a curve at a point." Do NOT connect to your curve here.
5. For your curve, adapt the Maple file diffgeomproj1.mw—at the bottom of the file, I have listed parameterizations and domain values for your curve. Move the commands for your curve to the top of the file, just under the packages, and delete the others. Follow the instructions and execute the rest of the commands to obtain output for your curve. You will collate the Maple output for your curve into the PDF you submit, so be sure to save any modifications.

6. For arc length of your curve, does Maple compute the arc length to obtain a specific value (closed form solution), represent the arc length as an elliptic integral or similar, or just spit back the arc length as an integral without any additional information at all? Be sure that you have executed the packages before responding to this question.

7. Starting with the parametrization of your curve $\alpha(t)$, compute T by hand, simplify if possible, and then compare and contrast with Maple's output. Show all by-hand work for the steps, derivatives and any simplifications and explain in words the similarities or differences with Maple's output.

8. The fundamental theorem of curves tells us that up to rigid motion in space, like rotation, translation and reflection, κ and τ define the curve via the Frenet-Serret equations for TNB derivatives. If we have an initial condition for $\alpha(0), T(0), N(0)$ and $B(0)$ then the curvature and torsion give a unique curve in space, so exploring the curvature and torsion in depth is one way to understand a curve.

   For curvature of your curve,

   (a) What kind(s) of curvature is possible on your curve (positive, negative, zero)? Approach this analytically from the output Maple provides for curvature as well as numerically using at least one evalf command. Explain/show reasoning.

   (b) Is 0 curvature or curvature close to 0 ever possible? If so, for what t values?

   (c) Does κ ever get large? If so, for what t values?

   (d) Discuss curvature **intuition** for your curve—connect the kinds of curvatures and radii of the osculating circles to the visualization of your curve, in terms of how curvy or flat it is in parts. You might include some related sketches too.

9. For torsion of your curve,

   (a) What kind(s) of torsion is possible on your curve (positive, negative, zero)? Approach this analytically from the output Maple provides for torsion as well as numerically using at least one evalf command. Explain/show reasoning.

   (b) Is 0 torsion or torsion close to 0 ever possible? If so, for what t values?

   (c) Does τ ever get large? If so, for what t values?

   (d) Discuss torsion **intuition** for your curve—connect how quickly or slowly the curve is leaving the osculating plane, if it is, to the visualization of your curve. You might include some related sketches too.

10. Next, read the sources I provide for your curve, accessible from
    `https://www.appstate.edu/~greenwaldsj/class/4140/project1links.html`
    Take from the sources I provide for you and perform additional research as needed for historical mathematicians, physicists, engineers, or others who are related to your curve. These can be people who laid groundwork on the curve or conducted peripheral but connected work. If you can't find historical people, more recent ones are OK too. List their full names.

11. Search for additional information on one person in the last question (or more than one if you prefer), like in MacTutor. If your curve is named for someone, research that person. Regardless, try to select a person with a substantial contribution and connection to your curve. Report back here and keep track of your sources for #15. For instance, perhaps you can find and summarize

    - their contributions or connections to your curve

- the title of their publication that included content related to your curve or a year or a range of years they worked on your curve, if possible. Or if not, then their year of birth and, if applicable, death, would provide their working years
- what country they worked in
- something you found interesting about the person

12. Search MathSciNet `https://library.appstate.edu/find-resources/databases/subject/mathematical-sciences` for a journal article related to your curve. Choose one you find interesting and write down the full bibliographic reference from the MathSciNet database. If you can't find anything on MathSciNet, it may be that your curve is more relevant in applications, so you can try another database or Google Scholar.

    What is MathSciNet? Historically, mathematicians communicated by letters, during visits, or by reading each other's published articles or books once such means became available. For example, Marin Mersenne had approximately 200 correspondents. Some mathematical concepts were developed in parallel by mathematicians working in different areas of the world who were not aware of each other's progress. In an effort to increase the accessibility of mathematics research articles, reviews began appearing in print journals like Zentralblatt fur Mathematik, which originated in 1931, and Mathematical Reviews, which originated in 1940. Since the 1980s, electronic versions of these reviews have allowed researchers to search for publications. MathSciNet, the electronic version of Mathematical Reviews, is "the authoritative gateway to the literature of Mathematics" and currently contains over 4 million items.

13. In bullet point format, summarize the *significance* of your curve in historical and current research, including (if possible) real-life applications or connections as well as the earliest year you can find related to your curve.

14. In bullet point format, summarize the physically interesting features of your curve.

15. Give proper credit to the links I provided for you as well as any other references you used, including proper credit to image sources. This includes citing sources professionally, including author names, except authors are not needed for images.

16. Collate your work into one PDF for submission to the ASULearn assignment for Project 1. **If you are a graduate student**, collate the graduate problems too. Electronically, you can append PDFs you create from Maple to the end of your other PDFs, like by using Preview on a Mac or PDFsam on a PC. Maple lists instructions for creating a PDF. If you have a phone or tablet, apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help.

17. **Elevator pitch presentation about your curve**—up to 100 seconds. The idea of an elevator pitch is to make a short, persuasive pitch that sparks interest in a topic during the time it takes to ride an elevator with a stranger. You will present your elevator pitch during class and each person is limited to 100 seconds. Start your pitch with your name and your curve's name. It's an elevator so your contribution is solely oral speech but I will project your curve, it's parameterization and an image of it to give the class something to refer to.

    If you work in a group then you should try and pitch different information than your partner did. You could pitch separately or take turns within the same pitch, but both people must speak about equally—as a group you are limited to 200 seconds total.