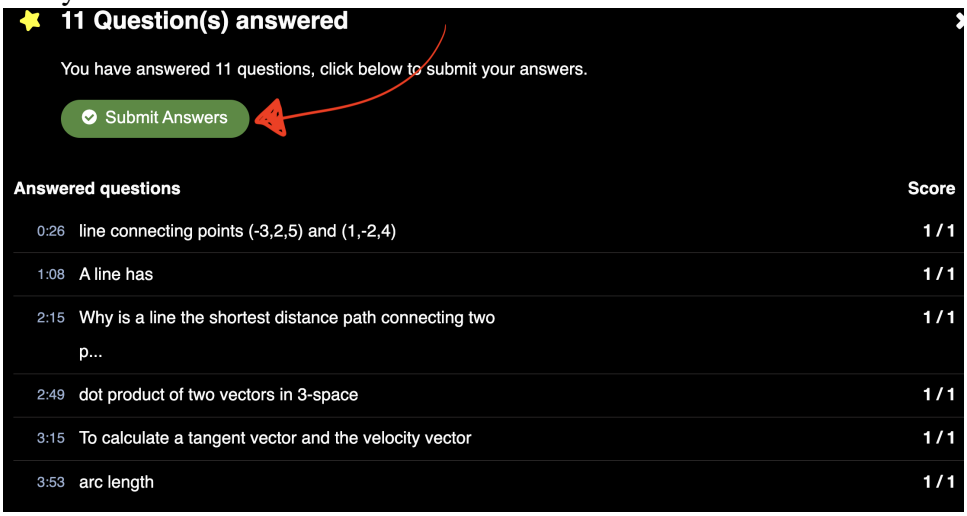
Differential Geometry Video Interactions

Dr. Sarah

These are interactions in some of the interactive videos I created. In the interactive video, the video pauses, asks a question, and requires a response to proceed. To earn credit we watch the entire video and submit the correct answers via the green "Submit Answers" button at the very end of the video, the one that shows all the questions we have answered—we use the check feature on interactive questions in order to help and can redo the responses until they are correct.



Each video includes directions "to pause regularly to take notes that you can bring with you to class especially on concepts, proofs, Maple, and other visualizations, and any remaining questions." Above each video we include resources needed such as Maple files to explore.

course intro interactive video

- What should you call me in class and in your electronic salutations (like the forum)?
- Is a growth mindset an important part of class?
- Open up course calendar above the video but still inside this activity.
- Even through there are strict deadlines, where is flexibility built in to the course?
- Consider the deadlines and activities and assignments on the PDF calendar.
- Consider the active learning environment and whether you can bring a laptop, tablet, or phone with you to classes. Notice that as shown in the picture, tables of four people are working together during class and it will help if some of you have a laptop with Maple (it's free) on it, or a tablet or phone you can bring to classes to access the internet.
- Consider the welcoming environment during class.
- Our hybrid class is officially designed by the registrar and scheduled by the university for our third hour's time in and out of class to be a part of the activities between classes. The university recommends 2-3 hours outside of class for each credit hour (c.h.) and we add the 3rd hour's time in too: that gives 2*3c.h. + 1 to 3*3c.h. + 1 as weekly engagement time outside of our Walker Hall classes. Since there are two of those, divide these ranges total by 2 to see the university recommended hours between each class.
- Where do we find the completion items and how do we know when they are completed?
- Are video notes important?

- How do we earn completion for interactive videos?
- What are some of the features of the interactive videos?
- Is group work allowed on projects?
- Consider how you will create a full-size multipage PDF of your project work and Maple responses
- You might have some aspects that need improvement on a project. Where do you go to see my feedback?
- Consider the 2 in-class assessments and their due dates and the 4 projects and their due dates in the PDF calendar from above the video
- Consider the grades and flexibility that is still within boundaries: Even with strict deadlines, there is some flexibility built in and multiple pathways for success, e.g., there is one revise and reflect opportunity for one of the first three projects and one in-class assessment. Some completion items have strict deadlines and others have multiple opportunities but the lowest ones are dropped. Try to attempt videos and take video notes by the listed date whenever possible as the material builds on itself.
- Consider the prereq of Calculus with Analytic Geometry III and the coreq of Introduction to Linear Algebra. A list of topics we'll use is in the syllabus and first day handout. Mathlab or online sources are great for reviewing computations from those classes as we use them in class or you see them in readings.
- Where can you go for help outside of class?
- What is some common advice from prior students?

lines and Maple intro interactive video

- Write this in your notes and consider the reverse direction—how to show that if the acceleration is the zero vector $\vec{0}$, then we have a constant speed straight line.
- Sketch a picture in your notes that showcases the latitude and great circle paths on the sphere.
- Compute the dot product of $\begin{bmatrix}1\\2\\3\end{bmatrix}$ and $\begin{bmatrix}-3\\0\\1\end{bmatrix}$
- Notice that hypotenuse is $|\vec{u}|$ and the adjacent side is $\vec{u} \cdot \frac{\vec{v}}{|\vec{v}|}$ so what is $\cos(\theta)$?
- What is the derivative of the dot product of $f(x) \cdot g(x)$?
- What is the dot product of 2 vectors on the same line?
- Sketch the picture in your notes and at $p = \alpha(a)$ label the 2 vectors $\alpha'(a)$ and $\frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}$.
- Consider how the dot product definition, distribution, and derivative, derivative of a constant vector, and the fundamental theorem of calculus came in to rewrite the length of the line as integral $\alpha'(t) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}$ and then obtain the inequality from there.
- If you have access to Maple where you are, open up the .mw file above the video so that you can get used to these commands and handling the animations and graphs. If not, still continue and you can go back to do this when you do have Maple access.
- What should we do about Maple's forgetfulness and lack of forgetfulness?

- If you are in the Maple file, after clicking on the graph, test out the play button, the slider button, and moving the graph itself around too. Otherwise, do this later on.
- Consider why Maple's output is $< 2/3, 2/3, 1/3 >$ instead of $< 1, 1, 1/2 >$.
- What does * stand for in Maple?

tractrix interactive video

- Which of the following could represent the line between the points (-3,2,5) and (1,-2,4)?
- Which of the following related to velocity or acceleration applies to a line?
- Why is the shortest distance path in Euclidean space between two points P and Q the line $P + t(Q - P)$
- Which applies to the dot product of two vectors in three space?
- Which applies to a tangent vector and the velocity vector?
- Which of the following are true regarding arc length?
- Compute the derivative of the tractrix curve $(\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$ in your notes
- Work out the trigonometric/algebraic reduction details on paper that I just described.
- Compute the magnitude.
- What does this further reduce to? Write the details of the computation in your notes.
- Open the TractrixMaple.mw file from above the video and try it yourself.
- Write the rest of the computation in your notes.
- Open the TractrixMaple.mw file above the video and try it yourself, modifying the parameters to match the tractrix curve $x = \cos(\theta) + \ln(\tan(\frac{\theta}{2})), y = \sin(\theta)$ and $z = 0$ and modifying the domain like we had for Pi/2 to 2*Pi/3.
- If you would like to have the Project 1 PDF and Maple file open while I'm introducing them, you can access them under ASULearn activities for project 1
- Consider some of the significance and physically interesting features of the tractrix that I mentioned previously.

arc length, tangent, and physical attributes interactive video

- In your notes, write down the formula for calculating arc length as well as in your own words what the arc lengths gives us physically and geometrically.
- We have s as a function of t. Consider how to solve for t, the inverse function.
- Consider how $t(s)$ was substituted in for t in the curve $\alpha(t)$ to form the reparameterized curve $\beta(s)$.
- Given what we assumed for contradiction, what is the sign of $s(y) - s(x)$ here?
- Why is it helpful for $t(s)$ to pass the vertical line test?

- In your notes, write down how to calculate T as well as in your own words what it tells us physically and geometrically.
- In your notes, write down how to calculate velocity and speed as well as in your own words what the velocity and speed tells us physically and geometrically.
- In your notes, write down how to calculate acceleration as well as in your own words what the acceleration tells us physically and geometrically.
- In your notes, write down how to calculate jerk as well as in your own words what the jerk tells us physically and geometrically.
- Open up the Maple file above the video, tractrixmaple3.mw
- Execute the Student[VectorCalculus] package and other beginning commands
- Execute more commands and consider Maple's output
- Play the animation using the play button or the slider
- Why isn't the speed of the reparameterized curve 1?

TNB 1 interactive video

- In your notes, write down the curve $\alpha(t) = (r\cos(\omega t), r\sin(\omega t), 0)$
- Compute $\alpha'(t)$
- What is $|\alpha'(t)|$?
- Write the trig reduction of speed in your notes.
- Write T for this curve.
- In your notes, write down the formula for calculating T as $\alpha'(s)$ and $\frac{\alpha'(t)}{|\alpha'(t)|}$ as well as in your own words what T shows us physically and geometrically.
- Compute $\frac{T'(t)}{|\alpha'(t)|}$ to compute the curvature vector $\vec{\kappa}$
- What is the scalar κ?
- Write $N = \frac{\frac{T'(t)}{|\alpha'(t)|}}{\kappa}$
- In your notes, write down the general formulas for calculating N by way of the curvature vector, the speed, and the scalar curvature that we need all of here as well as in your own words what N shows us physically and geometrically.
- Take the dot product of $T = (-\sin(\omega t), \cos(\omega t), 0)$ and $N = (-\cos(\omega t), -\sin(\omega t), 0)$
- What is the dot product of a unit vector with itself?
- Consider the derivative of $T \cdot T = 1$?
- Write the proof in your notes that the derivative of any unit vector is perpendicular to the original
- Sketch a picture of the curve and one osculating circle in your notes

- What happens when κ is small?
- Compute the cross product of T and N here—set up i, j, k as row 1, T and N as the other rows of the matrix and take its determinant, or however you did it in Calc III.
- Why is $B = T \times N$ a unit vector?
- Write one of these arguments in your notes, write how to compute B, and write what it means physically and geometrically.
- Open TNB1.mw from above the video.
- Some computations may take Maple a really long time—for my PhD thesis, I was running simulations over the entire weekend in Maple! What should I do if Maple is taking too long with computations like arclength and I want to stop it?
- Try the Maple file yourself and play around some.

TNB 2 interactive video

- By hand, calculate T, N, κ, B, and τ for the curve $\alpha(t) = (\frac{5}{13}\cos(t), -\sin(t), \frac{-12}{13}\cos(t))$. My hint is to repeatedly apply that $\sin^2 t + \cos^2 t = 1$. For example, the magnitude of $\alpha'(t)$ reduces nicely!
- Why do we divide by $|\alpha'(t)|$ to compute T and the curvature vector and to find τ, the torsion?
- Compare your by-hand work with $B = (-\frac{12}{13}, 0, -\frac{5}{13})$ in the end. You should have obtained $|\alpha'(t)| = 1$, i.e. this is a curve parameterized by arc length!
- How did we show that T and N are perpendicular?
- What is $T'(s)$?
- In your notes, write the argument that $B'(s)$ has only an N component in the linear combination $aT + bN + cB$, i.e. no B component and no T component.
- In your notes, write the argument that $N'(s) = -\kappa T + \tau B$
- Rotate your right hand away from you with B =thumb, T =index finger, and N =middle finger, to internalize that when $\kappa > 0$ and $\tau > 0$, T moves toward N and B moves away from N.
- First, sorry, I just said τ instead of T for κT, so please forgive my slip! Next, Rotate your right hand away from you with B=thumb, T=index finger, and N=middle finger, to internalize that when $\kappa > 0$ and $\tau > 0$, N moves away from T and toward B.
- See what Myka from *Warehouse 13* says and how it relates to curves.
- Open 4140TNB2.mw from above the video
- Hit enter to execute the Maple commands, including the Student[Vector Calculus] and plots packages. You can reveal the package contents with a ";" rather than ":" if you like.
- Examine Maple's output of speed as $\sqrt{2}\sqrt{\frac{1}{t^4+1}}$. What happens as t gets large?

- In the TNB animation command, Modify frames=20 to frames=100 and play the animation via the play button or the slider that comes up after you click on the animation visualization. You can also modify t1 and t2, like range= -15..15 instead of range=t1..t2, or similar, and you can also modify the width of the TNB vectors via [width=.02]. Depending on your modifications, you may notice that Maple sometimes has problems rendering the curve away from the broken part.
- Where does the metric slow down
- Where does the osculating circle get smaller (i.e. the —curvature—, 1/radius, get larger)?
- How can the mystery curve be a circle in a plane when z is not 0?
- Compare your by-hand work from the beginning of this video on T, N, κ, B, and τ for the curve $\alpha(t) = (\frac{5}{13}\cos(t), -\sin(t), \frac{-12}{13}\cos(t))$ to Maple's output.

curvature and torsion applications 1 interactive video

- If you don't already have these TNB formulas in readily accessible form for further use today, copy them down.
- What is $T'(s)$?
- Write the $\kappa = 0$ is a line argument in your notes.
- What is $B'(s)$?
- What will we do with $(\alpha(s) - \alpha(0)) \cdot B$?
- What is $T \cdot B$?
- Write the $\tau = 0$ means we stay inside the osculating plane argument in your notes.
- What is $\alpha(s) + \frac{1}{\kappa}N(s)$?
- Consider what are $\alpha'(s)$ and $N'(s)$.
- Write the $\alpha(s)$ is planar and has constant positive curvature then we are on a circle argument in your notes.
- What is $-\kappa T + \tau B$?
- Consider interpreting each term of $\omega(s)$ physically and geometrically.
- In the case of a Ferris wheel (named for a civil engineer!), how can we interpret $\omega(s)$?

curvature and torsion applications 2 interactive video

- Review the formulas we've been working with
- Substitute $T'(s)$ and $B'(s)$ into $\cos(\theta)T' + \sin(\theta)B'$
- What happens when we cross multiply from $\frac{\cos(\theta)}{\sin(\theta)} = \cot(\theta) = \frac{\tau}{\kappa}$?
- I left out some steps to fit it on the slide. Write the full argument that $\alpha(s)$ with $\frac{\tau}{\kappa}$ constant and κ nonzero is a cylindrical helix

- If z points straight up, then where is the annulus in the 4 images?
- Compute $\alpha'(t)$ and then $|\alpha'(t)|$
- Consider the computations for the curvature vector
- Sketch the strake and the annulus. Label the inner helix curve on the strake and the inner circle on the annulus in the pictures.
- Write the parametrizations of the inner helix on the strake and the inner circle on the annulus
- Write the parametrizations of the outer helix on the strake and the outer circle on the annulus.
- open curvaturetorsionapps.mw in Maple
- consider velocity, acceleration, jerk, speed, arc length, curvature, and torsion for this inner helix on the strake
- consider T, N, and B for this inner helix on the strake
- play the animation for the T, N, B frame for the inner helix on the strake
- play the animation for the osculating circle. What do you notice?
- How do the curvatures compare?
- The inner circle on the annulus matches the length of the inner helix on the strake, so we already have that from Maple, but why do we need a proportion for the outer circle on the annulus?

surfaces, geodesics and coverings interactive video

- What role does the r parameter play here?
- Write the parametrization of the strake and the values of the parameters.
- What is the dimension of a surface, the smallest number of parameters we need to represent the space?
- Consider these questions for geodesics on the cylinder: 1. Can a geodesic ever intersect itself? 2. How many differently shaped geodesics can you find? What do they look like? 3. Is straight always shortest distance? 4. Is shortest distance always straight? 5. How many geodesics join 2 points?
- Sketch pictures in your notes
- Consider locally shortest distance versus intrinsically straight
- Fold a paper in half vertically so you have 2 equal regions. Label point A on each edge at the same height (3 As) Choose Bs not on the same vertical or horizontal line as A. Draw a line between every A and every B. Marker is best. Roll the sheet up so As match and examine the geodesics
- How many different geodesics are between A and B shown here in this 2-sheeted cylinder covering?
- How about if we added a 3rd sheet to the cylinder covering? How many different geodesics would we have now between A and B?
- Sketch a picture in your notes

- Sketch a picture of the cone and the covering and label $\theta - \beta$ to show the secant relationship.

coordinates and geodesic curvature interactive video

- Consider the Calculus with Analytic Geometry III connections
- What is $x^2 + y^2$?
- What is the coordinate curve $x(\theta, z)$ with z constant on this cylinder?
- What is the coordinate curve $x(\theta, z)$ with θ constant on this cylinder?
- Compute x_z
- Compute U
- Sketch a picture of the cylinder with the coordinate curves at a point, x_theta and x_z tangents to those coordinate curves, the tangent plane, and U
- Substitute $\theta = \frac{s\cos(\alpha)}{r}$ and $z = s\sin(\alpha)$ into the cylinder $(r\cos(\theta), r\sin(\theta), z)$ to see how these kinds of substitutions work to create a curve as a function of only s on the cylinder here (as α and r are constant). Write this in your notes as we'll be coming back to it again later.
- Write the definitions of the unit normal U, the normal curvature $\vec{\kappa}_n$, and the geodesic curvature $\vec{\kappa}_g$
- Write the computation steps of the curvature vector of the helix $(\cos(t), \sin(t), t)$ on the cylinder of radius 1.
- Consider why the helix$(\cos(t), \sin(t), t)$ means that $x(u, v) = x(t, t)$ on the $(\cos(u), \sin(u), v)$ cylinder
- Consider how the curvature vector is completely normal to the surface, so not felt by the bug
- Play around some in this Maple file above the video to get used to it! Change g and the curve commands to match, remove items in the second display command, rotate the plot...
- Modify c2 to 20 to extend it, modify the parameter functions in Maple: f1 to t*cos(Pi/16) and f2 to t*sin(Pi/16), return there, and then return below in the plotting commands.

speed of a geodesic interactive video

- Consider driving a car along these curves. Do you have to turn the steering wheel?
- Write the argument that the speed of a geodesic is constant.
- Does a geodesic has to have constant speed? Does a curve with constant speed have to be a geodesic?
- Review how we computed the normal to the surface and consider the normal to this unit cylinder $x(u, v) = (\cos(u), \sin(u), v)$.
- Review how we computed the normal to the surface and consider the normal to this unit cylinder $x(u, v) = (\cos(u), \sin(u), v)$.
- Compute the speed of the curve $(\cos(t), \sin(t), \sin(t))$
- Consider why the curvature vector to the curve $(\cos(t), \sin(t), 0)$ is completely normal to the unit cylinder

- What is the curvature vector to the curve $(\cos(0), \sin(0), t)$
- Write this argument about classifying the geodesics on a cylinder analytically
- How about $v = 0$ as one example of a u coordinate curve on the geographical sphere. What is that?
- Sketch some pictures in your notes that show some of the latitudes for the u coordinate curves when v is fixed and some of the longitudes for the v coordinate curves when u is fixed.
- Write the computation for U on the geographical sphere in your notes
- Open the Maple file above the video
- Try some modifications yourself. Plug in other functions of t for f1 and f2.
- Overall, what did our examples show?
  a) geodesics must have constant speed and the geodesic curvature vanishing
  b) curves that aren't geodesics may have constant speed but they won't have vanishing geodesic curvature
  c) both of the above
  d) neither
- Write this part of the argument
- Consider this argument that geodesics on the sphere are great circles and write the rest of it.

first fundamental form interactive video

- Write this argument of how E, F and G and the metric form arise from our usual definition of arc length along a curve as well as the definitions.
- Write these computations down for the metric form for the $(u, v, 0)$ plane and the $(u\cos v, u\sin v, u)$ cone.
- Consider the roles of the u and v parameters on the new plane $(\sqrt{2}u\cos\frac{v}{\sqrt{2}}, \sqrt{2}u\sin\frac{v}{\sqrt{2}}, 0)$ and the curves they generate when one is fixed and the other varies
- Compute E, F and G on the new plane $(\sqrt{2}u\cos\frac{v}{\sqrt{2}}, \sqrt{2}u\sin\frac{v}{\sqrt{2}}, 0)$
- Open the Maple file from above the video and execute the new plane and cone commands by hitting return and compare with your by-hand work
- Why is F repeated as the off-diagonal of the matrix, giving us a symmetric matrix where the transpose of the matrix is the same as the original?
- Write this argument of how g_{ij} as an inner product determines dot products of tangent vectors as well as angles. At the same time, review the multiplication of 3 matrices if that feels rusty
- Which are true about the intrinsic versus extrinsic nature of the normal curvature and geodesic curvature?
- Compute E, F and G on the geographic sphere and write the metric form in matrix notation
- What is the implication of the metric form on the sphere?
- Are the catenoid and helicoid locally isometric?

- Are the saddle and Enneper's surface locally isometric?

shape operator interactive video

- Sketch the top left picture in your notes and label $p, U, w, Sp(w)$ and $\nabla_w U$
- Sketch the shape operator on the cylinder from the middle picture and write the shape operator computations for the cylinder
- Open the Maple file from above the video and execute the initial commands and commands for the sphere by hitting return.
- For the geographical sphere, what do you notice about $-U_v$ as a linear combination of x_u and x_v?
- Compare the last coordinate of x_u with the last coordinate of $-U_u$ for the pseudosphere
- For the pseudosphere, consider how the first coordinate of $-U_u$ is $-\frac{1}{\sinh u}$ times the first coordinate of x_u
- What are the augmented matrices to solve for $-U_u$ and $-U_v$ as linear combinations of x_u and x_v?
- Write the shape operator for the pseudosphere
- Does $-U_u$ in the shape operator always have a vanishing x_v component of 0?
- Is the x_v component of $-U_u$ for the shape operator always equal to the x_u component of $-U_v$, the mixed components?

second fundamental form and Gauss's Theorema Egregium interactive video

- Compute the quantities listed for the geographical sphere and write them in your notes.
- Take the derivative of $U \cdot x_u = 0$ with respect to u and write it in your notes
- What should we take the derivative of to obtain m?
- Compute l for the geographical sphere and reduce.
- Compute the shape operator for the round donut via $-U_v$ and writing that as a linear combination of x_u and x_v.
- If we have a surface $x(u, v)$ and we hold u constant, then $x(constant, v)$ will be a...
- Which are geodesics on a round donut?
- Which are geodesics on a flat torus?

surface area and computations on the strake, cylinder, sphere and cone interactive video

- Write the proof that the square root of the determinant of the metric form gives the area of a flat x_u, x_v parallelogram
- Extend our prior computations for the torus. Which is the surface area of a round torus?
- Open the Maple file for the strake computations, hit return to execute them, and then consider the outputs.

- Compute and write the cylinder computations as practice
- Why is an improper integral needed for the sphere and cone surface area?

Gauss-Bonnet interactive video

- Is the Gauss curvature of a horse saddle negative?
- What is the Gauss curvature of a sphere of radius r?
- What is the total Gauss curvature (the double integral of KdA) of a sphere of radius r?
- What is the Euler characteristic of a cylinder?
- A triangulation of a surface breaks it up into triangles, which allows us to compute the Euler characteristic. Which can we apply to the sphere?
- What is the sum of the angles in a spherical triangle?
- What is the sum of the angles in a triangle on a flat torus?
- Which is true about the Klein bottle parameterizations?

surfaces not embedded interactive video

- Which cannot be C^2 embedded in $\mathbb{R}^3$ from among hyperbolic geometry, stereographic projection of the sphere, flat torus?
- Sketch a rough picture of the annular model and label δ, w, and s.
- Sketch a picture of a flat annulus that includes a center point, an inner radius of r and outer radius of $r + \delta$.
- Write out the proof in your notes.
- Compute K and write down the details.
- Write out an argument that the angle sum of a geodesic triangle is less than π for hyperbolic geometry.
- Solve the double integral for the surface area of a region bounded by a horocycle and radial geodesics d apart by hand and write out the details in your notes.

geodesic equations, tensors, and spacetime interactive video

- For which surface did we use covering arguments?
- Where did we use a symmetry argument?
- Why is x_u, x_v, U a basis for a regular surface?
- What does the dot notation mean above a vector?
- What is $x_u u \cdot U$?

- Consider how to write x_{uv} in the basis with coefficients from Christoffel symbols and the second fundamental form.
- Consider the xv components of $\ddot{\alpha}$
- Why would a geodesic have zero x_u, x_v components of $\ddot{\alpha}$?
- Consider how the Einstein summation notation is equivalent to the forms above it.
- Consider E_u.
- What happened to the other components of x_{uu}?
- Why do we need a symmetric matrix for g_{ij}?
- What is the Christoffel symbol Γ^u_{uu} for a surface?

Minkowski spacetime and Christoffel computations interactive video

- Why is there no $dt\,dx$ term?
- Consider what is $(\frac{ds}{dt})^2$?
- What kind of relativity is Minkowski's space?
- Write out the matrix multiplication.
- Consider what surface $x^2 + y^2 = t^2$, where t varies too.
- What is the geometry of the set of v's satisfying $|v| = 0$ in Minkowski space?
- Consider the punchline.
- This should look familiar. What surface is this?
- What is the second coordinate?
- Consider what is the matrix representation of this metric?
- Consider $a = 1$ in the Euler-Lagrange equation and how the partials and the derivative with respect to arc length in part c) applied to I in part b) reduced in part d).
- Write out a) through e) in your notes.
- Which Christoffel symbol is not 0?
- Repeat c)—f) but with $a = 2$ for the θ coordinate.
- Which Christoffel symbol is not 0 for $a = 2$?

wormhole metric and relativity interactive video

- Consider the last two terms of the wormhole metric, which should be familiar. What are they from?
- How many Christoffel symbols should we have for spacetime?

- Compare your notes on Γ^2_{12} from our prior work on the Christoffel symbol of the sphere to Γ^4_{43} here.
- How many entries does the scalar curvature have?
- On the left side of this picture is a superhero outfit. What would GR stand for?
- If you haven't already done so, use the ASULearn choice selection feature to choose a metric for Project 3.

general relativity and the field equations interactive video

- What is the force on m? Click true when ready.
- Take the partial of ϕ with respect to r.
- Take the partial of r with respect to x.
- Use the quotient rule on $\frac{Mx}{r^3}$ to take the partial with respect to x.
- What role do the Christoffel symbols play?
- What is another way to write Einstein's field equations?
- How many equations and unknowns do we have?
- What are the solutions to Einstein's Field Equations?
- What are some surfaces with Gaussian curvature constant?
- What is the hypersphere that Einstein presented in 1917 as a model for the universe that was consistent with relativity?