Exam 2: Surfaces

It is time for our second exam in order to be sure that everyone reviews some of the fundamental concepts before we move on to the geometry of space-time and relativity.

At the Exam

This is an individual exam. You are on your honor to work on this exam by yourself (see `https://academicintegrity.appstate.edu/`). You may not seek nor accept help from any person or source. Do not use books, notes, online resources, or similar, with the exception of the reference card below. You may not give help to others. Recall that you can correct your lowest exam, as per `http://cs.appstate.edu/~sjg/class/4140/examcorrections4140.pdf`.

- In advance, you may make yourself some reference notes on both sides of a 3"x2.5" (half of a 3x5 card) The mini reference card must be handwritten. Think of the card as a way to include some important concepts, computations, or derivations that you aren't as comfortable with. You won't have room for much, so you should try to internalize as much as you can.

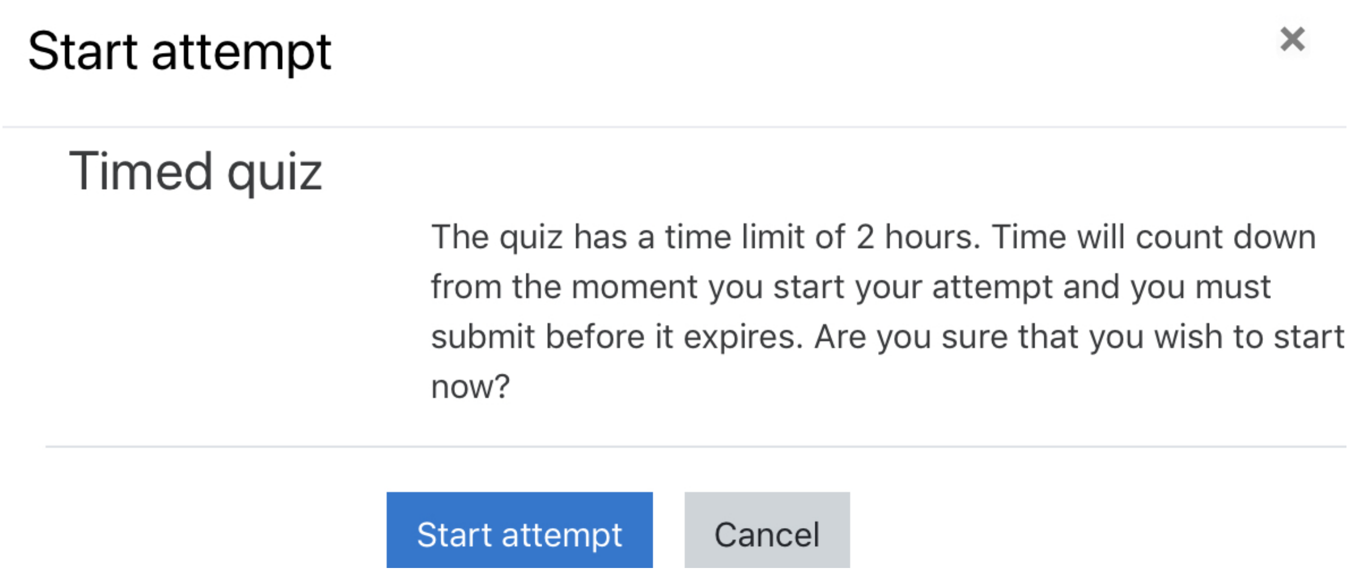
- You must handwrite your responses to the exam and scan them to a single PDF. Only a handwritten exam will be accepted.

- One scientific calculator or graphing calculator allowed. I don't see that you would need this, but I know some people like to have it with them.

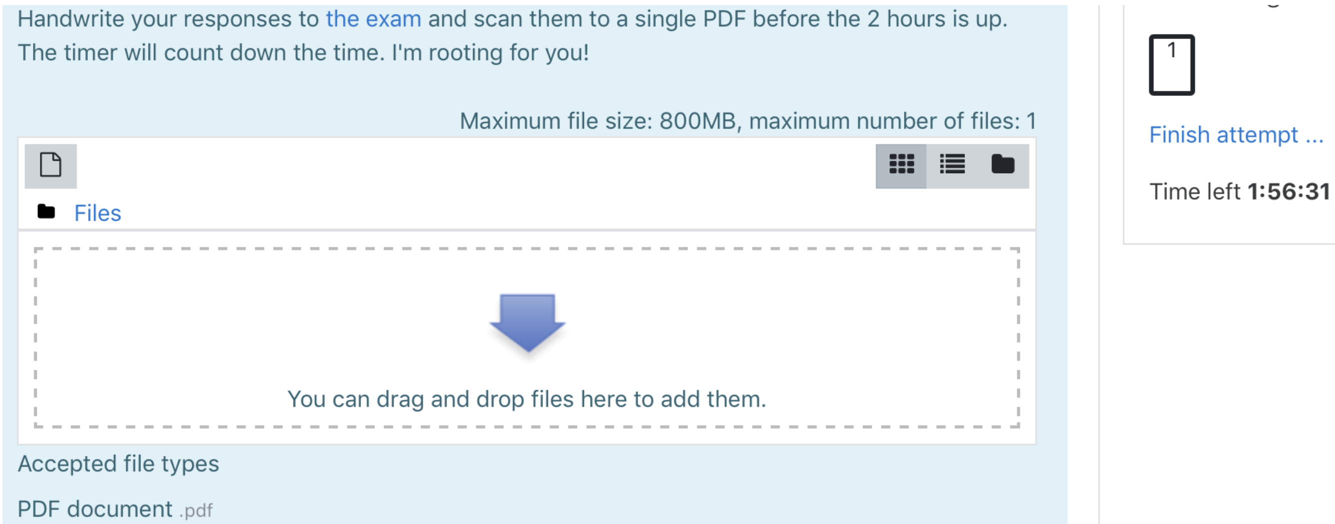
- You may have food, hydration, ear plugs, or similar if they will help you.

- Select a 2-hour block of time that is convenient for you between Thursday April 9th at 11am and Saturday April 11th at 11am. You can work in whatever block is convenient, but it must be consecutive minutes. Consider this as approximately 1:15 to take the exam, 5 minutes to download the exam (you may print it if you like), and the rest of the time to create and upload the single PDF of your handwritten work using a program like CamScanner or Genius Scan + or similar.

- When you are ready to begin, obtain the exam via "Attempt quiz now" and "Start attempt"



Then use control click on "the exam" (in blue below) to open or download the link and leave the timer on the right side open (if you need to, you can go back in to ASULearn and click again as long as time is still left).



- There will be various types of questions on the exam and your grade will be based on the quality of your responses in a timed environment.
  - Part 1: Fill in the blank/short answer
  - Part 2: Calculations and Interpretations
  - Part 3: Short Derivations/Proofs
- Partial credit will be given, so (if you have time) showing your reasoning or thoughts on questions you are unsure of can help your grade.

Review Surfaces

I suggest that you review your class notes on surfaces, the related calendar and class activities page (that has the slides and clicker questions), and ASULearn activities and solutions on the Theory of Surfaces.

Part 1: Fill in the blank/short answer There will be some short answer questions, such as:

- definitions related to any of the items in the glossary on surfaces
  `http://cs.appstate.edu/~sjg/class/4140/glossarysurfaces.pdf`
- parameterizations, shape and geometric properties of these 9 surfaces: catenoid, cone, cylinder, helicoid, hyperbolic plane, plane, sphere, strake, torus
- symbols, formulas, or physical/geometric descriptions like those on the matching activity
  `http://cs.appstate.edu/~sjg/class/4140/tactivitiessurfaces1.pdf`
- questions similar to previous clicker questions, the matching or other activities from class or homework where you fill in a blank instead. For instance,
  - $E =$ ______________________
  - $\vec{x}_u \cdot \vec{x}_u =$ ______________________

- ○ normal curvature components of a curve = ______________

  As you can see here, there is often more than one answer possible for fill in the blank questions: choose one response. Full credit responses demonstrate deep understanding of differential geometry. For instance, here you could fill in $E = \vec{x}_u \cdot \vec{x}_u$ for the first response, $\vec{x}_u \cdot \vec{x}_u = E$ for the second, and normal curvature components of a curve $= (U \cdot \vec{\kappa}_\alpha)U$ for the third, among other possible responses. On the other hand, responding with $E = E$, while a true statement, doesn't demonstrate deep understanding of differential geometry. Informal responses are fine as long as they are correct and demonstrate understanding of the material from class and homework.

Part 2: Calculations and Interpretations There will be some by-hand computations and interpretations, like

- Finding $\vec{x}_u$ and $\vec{x}_v$ for a surface
- For the plane and the cylinder (where the computations are quicker), finding the curvature vector of a curve $\vec{\kappa}_\alpha = \dfrac{T'(t)}{|\alpha'(t)|}$, a unit normal to a surface $U = \dfrac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$, the normal curvature $\vec{\kappa}_n = (U \cdot \vec{\kappa}_\alpha)U$ and the geodesic curvature $\vec{\kappa}_g = \vec{\kappa}_\alpha - \vec{\kappa}_n$ (what curvature is left over, if anything, for the bug to feel) for a curve.
- Interpreting whether curves are geodesics via
  a) a given geodesic curvature
  b) a covering argument, if one applies
  c) geometric/physical or other arguments such as those relating to symmetry of feet
  d) geometric/physical arguments about whether the curvature vector is completely in the normal direction or not. [For example, for a circle on a surface, we know the curvature vector of any circle points to the center of the circle. Combine this with intuition about the normal to a surface to say whether the curvature vector is parallel to the normal and hence gives a geodesic (or not)]
- Finding E, F and G and interpreting whether the Pythagorean theorem holds ($E = G = 1, F = 0$) or not, or whether $\vec{x}_u$ and $\vec{x}_v$ are perpendicular ($F = 0$).
- For the plane and the cylinder (where the computations are quicker), computing I, II, K, and the shape operator
- Interpreting selections from Maple files that you've seen in class and homework.
  a) Maple file on geodesic and normal curvatures
  `http://cs.appstate.edu/~sjg/class/4140/curvature2.mw`
  b) Maple file on Applications of the first fundamental form (isometries and area)
  `http://cs.appstate.edu/~sjg/class/4140/firstfundform.mw`
  c) Maple file on strake and I, II and K
  `http://cs.appstate.edu/~sjg/class/4140/strake.mw`

  For example, I could present you with some code and/or Maple output and ask you questions, like what does this show us (the helicoid is isometric to the catenoid, or this is not a geodesic, for example), or ask you to fill in the visualization (sketch the curvature vector to a curve, normal curvature, and geodesic curvature)

- Given K, interpreting what it tells us intuitively about the surface and being able to roughly sketch (for $K < 0, K = 0$ and $K > 0$)

Part 3: Short Derivations/Proofs Review and be able to prove the following:

- Prove how E, F, and G and the metric equation arise from our usual definition of arc length along a curve.

  slides 1–6: `http://cs.appstate.edu/~sjg/class/4140/coneandplane2.pdf`

- Prove that a geodesic must be a constant speed curve.

  slides 1–5: `http://cs.appstate.edu/~sjg/class/4140/geospeed.pdf`

- Prove how g_{ij} determines dot products of tangent vectors

  slides 7–11: `http://cs.appstate.edu/~sjg/class/4140/coneandplane2.pdf`

- Prove that the determinant of the metric form gives the area of a flat $\vec{x}_u, \vec{x}_v$ parallelogram

  slides 1–7: `http://cs.appstate.edu/~sjg/class/4140/surfaceareal.pdf`

- Prove $S(\vec{x}_u) \cdot \vec{x}_u = \vec{x}_{uu} \cdot U$ (i.e. how l was derived)

  slides 4–7: `http://cs.appstate.edu/~sjg/class/4140/Gausstheoremaegregiuml.pdf`

- In hyperbolic geometry with radius r as the interior radius of the annuli of width δ, with δ approaching 0, prove that if two geodesics are d units apart along the base curve and we travel c units away from the base curve along each geodesic, then they are at a distance of $d \exp(\frac{-c}{r})$.

  slides 9–16: `http://cs.appstate.edu/~sjg/class/4140/hyperbolicl.pdf`

We covered the proofs above in classes, as in the slides above. You should know the results of other statements, which could be asked about in the first two sections of the exam, but I won't ask you for any other complete proofs, other than the ones above.

I want you to understand and I am happy to help!