Homework 2: Curves

You may work alone or in a group of at most 2 people, and turn in one per group. You should be prepared to present any of the problems your group turns in. Show reasoning and work—each group must write up their work in their own words. You do not need to simplify your answers, but you do need to show by-hand computations. Try all the problems, but carefully write up to turn in this number:

4140 students: turn in 5 of the first 6 problems. Graduate students: turn in all 7 problems. The Maple file listed below is on the class webpage.

The purpose of homework is to learn and practice computational strategies, concepts, and develop critical thinking and problem-solving skills, so you should try problems on your own. Feel free to talk to me or each other if you are stuck on this assignment, but be sure to acknowledge any sources other than your group members or me, including other classmates like "The insight for this solution came from a conversation with Joel." If you use any external sources outside of your group then cite them. If you know how to do a problem and are asked for help outside of your group, try to give hints rather than the solution.

1. The sights of our everyday lives are filled with curves, many recognizable. You'll have to open your eyes for this problem. Find, either in person or in a picture, a curve which strikes a chord with you. (Circles and lines are unacceptable, be more creative.) Art, architecture, and nature are great places to search for resonating curves.

   (a) Describe where the curve arises or is located, and what significance it has.

   (b) Why is it personally meaningful?

   (c) Sketch it.

   (d) If you can parameterize it or find a parameterization of it, list it, and if not, briefly discuss what is challenging about that.

2. Choose your favorite planar curve. Write down a parametric version of the curve. Use the first part of the Maple file **homework2maple.mw** to find the Plot, Velocity, Acceleration, Jerk, Speed, ArcLength, Curvature, and Torsion. Rotate the plot to help you visualize it.

   (a) Print or sketch and write down what Maple finds.

   (b) Next provide a formula for calculating each term by hand (equation and/or words) and explain in your own words what each of the pieces means physically and/or geometrically.

   An example explanation for part (b) and velocity might be something like:

   > *The velocity is the first derivative of position componentwise (with respect to time) so if* $r(t) = (x(t), y(t), z(t))$ *then the velocity is* $(x'(t), y'(t), z'(t))$ *and it gives us a tangent to the curve at the given point. The velocity represents the way the position is changing as both a direction and a number, the speed, which is the length of the velocity vector. A tangent vector is the best fit line to a curve at a point.*

   You may use a web search and/or the book to help you but be sure to give proper credit.

3. Choose your favorite non-planar curve. Write down the parametric version of the curve. Use the first part of the Maple file **homework2maple.mw** to find the Plot, Velocity, Acceleration, Jerk, Speed, ArcLength, Curvature, and Torsion.

   (a) Print or sketch and write down what Maple finds.

   (b) What happens to the osculating plane—does it change or remain the same—and why? What does this tell you about the torsion?

4. Choose **one** of the following curves and enter it into the TNB Animation portion of **homework2maple.mw**. Play the animation and rotate it to help you visualize.

   Witch of Agnesi $(2t, \frac{2}{1+t^2}, 0)$
   Enter into Maple: (2*t, 2/(1+t^2), 0) with t ranging from -1.5 to 1.5

   OR Lemniscate of Bernoulli $(\frac{3\cos t}{1+\sin^2 t}, \frac{3\sin t\cos t}{1+\sin^2 t}, 0)$
   Enter into Maple: (3*cos(t)/(1+sin(t)*sin(t)), 3*sin(t)*cos(t)/(1+sin(t)*sin(t)), 0) with t ranging from -2*Pi to 2*Pi

   OR Vivani's Curve $(1 + \cos t, \sin t, 2\sin\frac{t}{2})$
   Enter into Maple: (1+cos(t), sin(t), 2*sin(t/2)) with t ranging from -2*Pi to 2*Pi

   (a) Sketch the curve and the Frenet Frame at two different places on the curve. Distinguish between T, N and B on your frame, either by color or by labels.

   (b) Next provide a formula for calculating each vector in the Frenet Frame (equation and/or words) and explain in your own words what each of the pieces means physically and/or geometrically.

   (c) Search for information about the curve in our book and on the web and summarize what you found in your own words.

5. Cycloid $\alpha(t) = (t + \sin t, 3 - \cos t, 0)$

   Enter the curve into the TNB Animation portion of **homework2maple.mw** as

   (t+sin(t), 3 - cos(t),0) with t ranging from 0 to 7

   Play the animation and rotate it to help you visualize.

   (a) Sketch the curve and the Frenet Frame at two different places on the curve. Distinguish between T, N and B on your frame, either by color or by labels.

   (b) Compute T by hand.

   (c) Is T in the Frenet Frame defined everywhere in this domain? If not, specify any problem points and explain.

   (d) Is B defined everywhere in this domain? Why or why not?

   (e) Why is the cycloid interesting from a physics standpoint? You may use a web search, but write this up in your own words and give any citations.

6. Spiral $\alpha(t) = (3\cos t, 3\sin t, \log t)$

   Enter the curve into the TNB Animation portion of **homework2maple.mw** as

   (3*cos(t), 3*sin(t),log(t)) with t ranging from .0000001 to 2*Pi

(a) Sketch the curve and the Frenet Frame at two different places on the curve. Distinguish between T, N and B on your frame, either by color or by labels.

(b) Compute T by hand.

(c) Is T in the Frenet Frame defined everywhere in this domain? If not, specify any problem points and explain.

(d) Why is a helical curve interesting from a physics standpoint? Use a web search, but write this up in your own words.

Graduate Problem: Computer Images of Curves

(a) In Maple, use a plot command to plot the planar curve $y = \sqrt{10^{-30} + x^2}$. When plotted from x=-1..1, the curve looks like it behaves the same as $y = \|x\|$ at the origin. Test out smaller ranges of x in the form of $\frac{1}{10...0}$(i.e. $x = -\frac{1}{10}..\frac{1}{10}$, etc). Can you distinguish the behavior of these curves at the origin by a similar Maple plot command? If so, what is the largest value of x of the form $x = -\frac{1}{100...0}..\frac{1}{100...0}$ that can distinguish them?

(b) Can you distinguish the behavior of the curves at the origin using differential geometry techniques? If so, explain how, and show work to distinguish the curves at the origin.