Homework 6: Research and Investigate a Surface

You may work alone or with one other persion & turn in one per group. Surfaces are on a **first come-first-served** basis in the ASULearn choice selection feature. If you are working in a group, one person selects the surface and the other selects "working with someone else who already selected our topic."

p. 74:

- 2.1.14 helicoid
- 2.1.16 Enneper's Surface

p. 80:

- 2.2.4 Mobius strip

p. 114-117:

- 3.2.11 hyperboloid of 2 sheets
- 3.2.12 hyperboloid of 1 sheet
- 3.2.13 elliptic paraboloid
- 3.2.14 hyperbolic paraboloid
- 3.2.16 saddle
- 3.2.17 Kuen's Surface
- 3.2.19 cone

p. 120-121:

- 3.3.2 torus
- 3.3.9 pseudosphere

p. 218 in the print book:

- color picture ellipsoid

p. 168-170:

- 4.2.8 Henneberg's Surface
- 4.2.8 Catalan's surface
- 4.2.8 Scherk's Fifth Surface

p. 197:

- 4.9.1 catenoid
- strake

If the book has a parameterization then you should use that one (see above). If not, you can find one on Wolfram's MathWorld. Other surfaces can be chosen if they have a reasonable parametrization that can be used for the Maple worksheets.

Explore the following questions via researching our textbook and elsewhere (**keep track of ALL your references for # 17**) and our Maple files. **Write it up in your own words** but you may use pictures from elsewhere (with proper reference). You will turn in all of the following and share some components with your classmates (see #18).

1. Provide or create a picture of the surface. Be sure to list any picture references (and any other references) in #17.

2. In bullet point format, summarize the physically interesting features of your surface.

3. Research two historical mathematicians who are related to your surface, including at least one mathematician from a country outside of the US when possible (it could be someone who laid groundwork on the surface or peripheral but connected work. If you can't find historical mathematicians, more recent ones are ok too.). In bullet point format, include relevant dates, names and their contributions to your surface.

4. In bullet point format, summarize the *significance* of your surface in historical or current research, including (if possible) real-life applications.

5. Search MathSciNet

   `https://library.appstate.edu/find-resources/databases/subject/mathematical-sciences`

   for a journal article related to your surface. Choose one you find interesting and write down the full bibliographic reference from the MathSciNet database. If you can't find anything on MathSciNet, it may be that your surface is more relevant in applications, so you can try another database or Google Scholar.

   What is MathSciNet? Historically, mathematicians communicated by letters, during visits, or by reading each other's published articles or books once such means became available. For example, Marin Mersenne had approximately 200 correspondents. Some mathematical concepts were developed in parallel by mathematicians working in different areas of the world who were not aware of each others progress. In an effort to increase the accessibility of mathematics research articles, reviews began appearing in print journals like Zentralblatt fur Mathematik, which originated in 1931, and Mathematical Reviews, which originated in 1940. Since the 1980s, electronic versions of these reviews have allowed researchers to search for publications. MathSciNet, the electronic version of Mathematical Reviews, currently contains over 4 million items.

6. Write general formulas for the following entities as a review, using letters, words, etc... Assume that you have a surface parametrized as $X(u,v)$ to start with. Do NOT do any calculations for your surface here, but do show generic formulas or explain how to calculate each from $X(u,v)$ itself (and your answers may build upon one another, ie using part a. in another part).

   (a) Normal to a surface $X(u,v)$
   (b) Curvature of a curve $\gamma(t)$ on $X(u,v)$
   (c) Normal Curvature of $\gamma(t)$ on $X(u,v)$
   (d) Geodesic Curvature of $\gamma(t)$ on $X(u,v)$
   (e) A curve is a geodesic if the geodesic curvature is...
   (f) E (of a surface $X(u,v)$)
   (g) F
   (h) G

7. Provide parametrization(s) for your surface that can be used for the Maple worksheets. If the book has a parameterization then you should use that one (see above). If not, you can find one on Wolfram's MathWorld.

8. Adapt the Maple file on geodesic and normal curvatures (`http://cs.appstate.edu/~sjg/class/4140/curvature2.mw`)

   to provide your new values for g and each of the following (these are the commands I used for a sphere or radius 1) for a curve that NOT is a geodesic. You may need to also modify other code, like to remove items in the display command to see the curvature vectors on their own, or change the limits on the coordinates for the tangent plane.

```
g := (x,y) -> [cos(x)*sin(y), sin(x)*sin(y), cos(y)]:
a1:=0: a2:=Pi: b1:=0: b2:=Pi:
c1 := 1: c2 := 3:
Point := 2:
f1:= (t) -> t:
f2:= (t) -> 1:
```

List what you used above include one or more pics that shows part of the surface, part of a NOT-a-geodesic curve, and the curvature vectors (label which is which).

9. Use the Maple file on geodesic and normal curvatures (`http://cs.appstate.edu/~sjg/class/4140/curvature2.mw`)

   in order to provide your new values for each of the following (these are the commands I used for a sphere or radius 1) for a curve that is a geodesic, , or as close to a geodesic as you can get:

```
g := (x,y) -> [cos(x)*sin(y), sin(x)*sin(y), cos(y)]:
a1:=0: a2:=Pi: b1:=0: b2:=Pi:
c1 := 1: c2 := 3:
Point := 2:
f1:= (t) -> t:
f2:= (t) -> 1:
```

   List what you used above include one or more pics that shows part of the surface, part of a geodesic curve, and the curvature vectors (label which is which).

10. Write general formulas for the following entities as a review, using letters, words, etc... Assume that you have a surface parametrized as $X(u, v)$ to start with. Do NOT do any calculations for your surface here, but do explain how to calculate each from $X(u, v)$ (and your answers may build upon one another, i.e. using part a in another part, and/or your answers in #6 above).

    (a) l

    (b) m

    (c) n

    (d) Gauss Curvature K of a surface $X(u, v)$

    (e) Mean Curvature of H a surface $X(u, v)$

11. Use the Maple file Gauss Curvature and Mean Curvature

    `http://cs.appstate.edu/~sjg/class/4140/gcandmchw.mw`

    to calculate

    (a) Normal to the surface

    (b) Unit Normal to the surface

    (c) E, F and G for your surface

    (d) l, m, n for your surface

    (e) Gauss curvature K for your surface

    (f) Mean curvature H for your surface

    If it is short enough, include what Maple provides. Otherwise include the beginnings of what Maple provides for each.

12. Write the metric form $(\frac{ds}{dt})^2$ for your surface (or the start of it if it is too unwieldy) and compare it to the flat Euclidean metric form. Does the Pythagorean theorem hold for your surface?

13. Set up, but do not solve, a surface area integral using E, F and G. Explain what your limits of integration would be to find the surface area for the entire surface. In addition, if your surface has infinite surface area, cap it off somewhere, and explain what the limits would be and what the capped picture would look like. [For instance for a parametrization of the xy plane $[u, v, 0]$, u and v range from $-\infty$ to ∞, but a capped version of the plane could be a square with u and v from -1..1]

14. What kind(s) of Gauss curvature is possible on your surface (positive, negative, zero)?

15. Discuss Gauss curvature **intuition** for your surface from the signs of the principal curvatures and which directions those curvature vectors point—refer to the visualization of your surface.

16. Use the Maple file Gauss Curvature and Mean Curvature

    `http://cs.appstate.edu/~sjg/class/4140/gcandmchw.mw`

    to explore the shape operator a bit.

    Is the rate of change of the surface normal U in the u or v directions a multiple of either $\vec{x}_u$ or $\vec{x}_v$ individually? Or does each application of the shape operator require a linear combination of both tangent vectors? Explain.

17. If you used any references other than our text, the Maple files or me, then give proper credit.

18. You will turn in all of the above as a single pdf on ASULearn under 4/2 hw 6 submit a pdf. You can use software like CamScanner or similar to create the pdf. If you are working in a group then one of you submits the pdf (be sure both your names are on it).

    In addition, prepare a short informal forum posting for your classmates based on the following components:

    - the title of your posting lists the name of your surface
    - introduce yourself and give your major(s)
    - a web address of a visual of your surface
    - one physically interesting feature from #2
    - one mathematician from #3 and their contributions to your surface
    - one real-life application or way your surface is significant from #4
    - whether the Pythagorean theorem holds for your surface from #12
    - Gauss curvature intuition for one interesting point on your surface from #15

    If you work in a group then you should try and post different information than your partner did. Submit it in the think-pair-share forum activity for #18 from hw 6.

19. Then respond to at least one of your classmates postings, who selected a different surface than you did, in a meaningful way. You might pose questions, extend ideas, or compare and contrast their surface to your surface. Another option would be for you to do some additional research on their surface and report what you found.