As connected to the homework readings, which of the following surfaces can not be embedded isometrically via a C^2 mapping into $\mathbb{R}^3$

a) hyperbolic geometry
b) stereographic projection of the sphere
c) flat torus
d) all of the above
e) exactly two of the above

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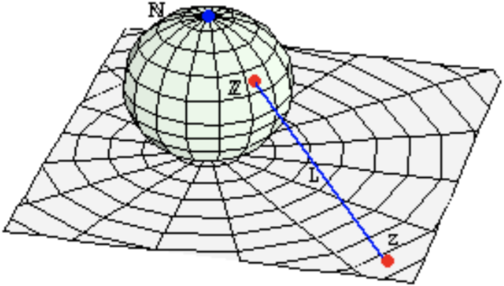
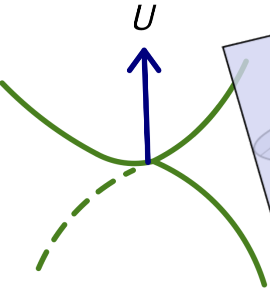
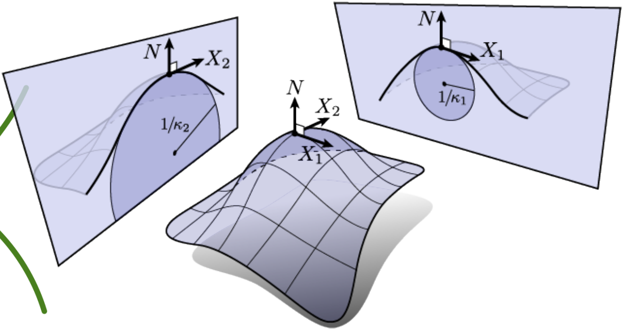


http://mathfaculty.fullerton.edu/mathews/c2003/complexfunreciprocal/ComplexFunReciprocalMod/Images/ComplexFunReciprocalMod_gr_44.gif

Constant Negative Gauss Curvature?

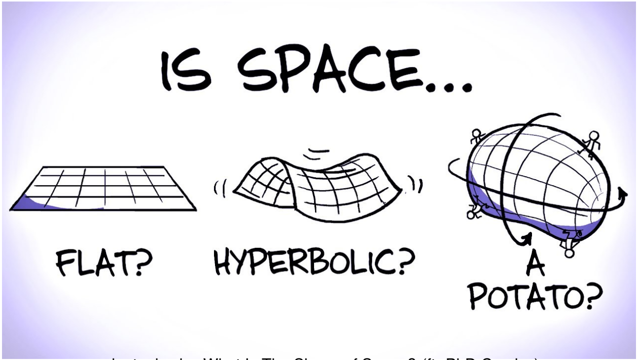




http://brickisland.net/cs177/?p=144

Hyperbolic Geometry

Does the real universe have curves?



minutephysics What Is The Shape of Space? (ft. PhD Comics)
https://www.youtube.com/watch?v=oCK5oGmRtxQ/

No not that geometry! (exaggerated, hyperbola),

Hyperbolic Geometry

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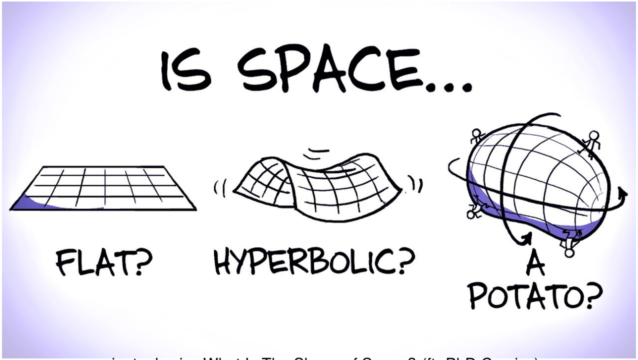

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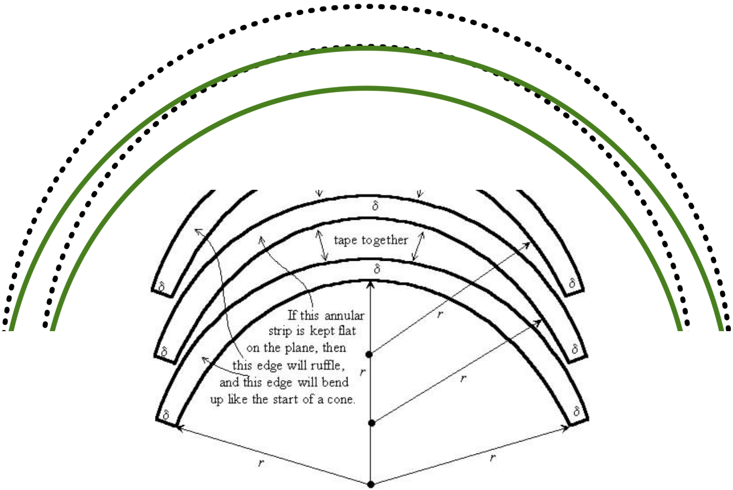
No not that geometry! (exaggerated, hyperbola), although is named for a hyperboloid because of a connection of one analytic model to it

Hyperbolic Geometry: Annulus Model



http://www.math.cornell.edu/~dwh/papers/crochet/38357f73.jpg

Annulus model is limit as $\delta \to 0$

- no extrinsic coordinates—no C^2 embedding into any $\mathbb{R}^n$!

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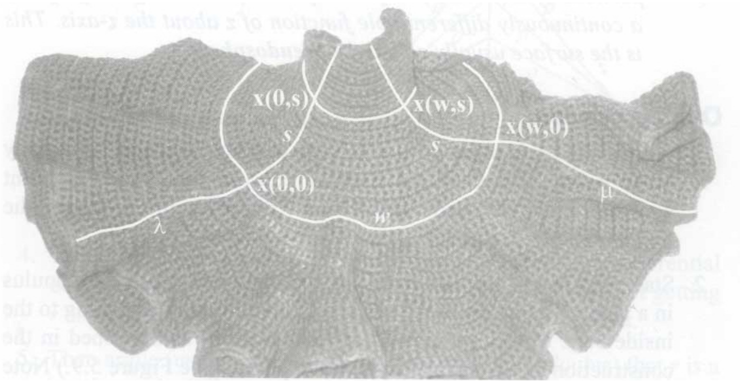
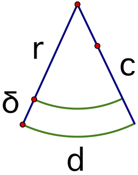


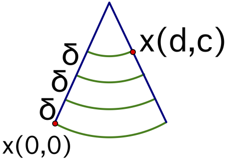
http://pi.math.cornell.edu/~dwh/papers/crochet/crochet.PDF

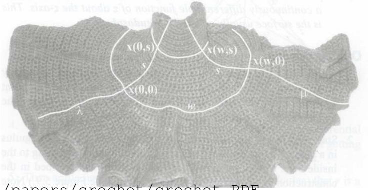
Annulus model is limit as $\delta \to 0$

- w–base curve horocycle
- s is $\perp$–λ, μ curves are (radial) geodesics by symmetry

Hyperbolic Geometry: Prove Distance is Exponential!







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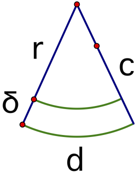

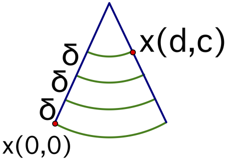

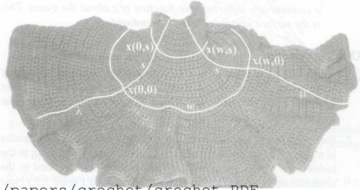

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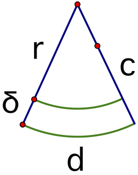

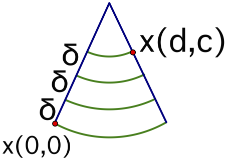

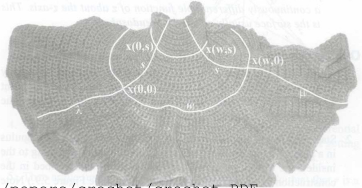

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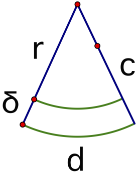

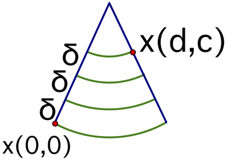

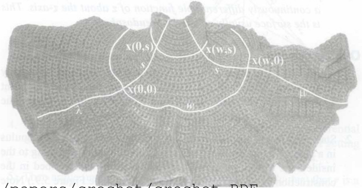

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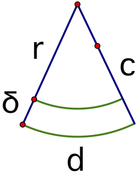

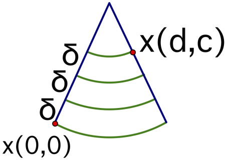

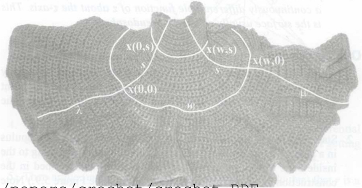

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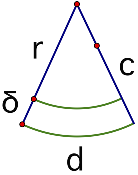

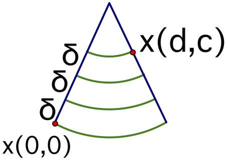

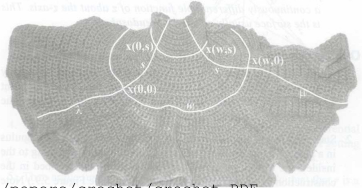

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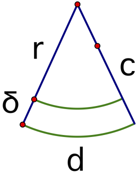

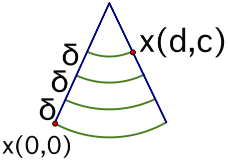

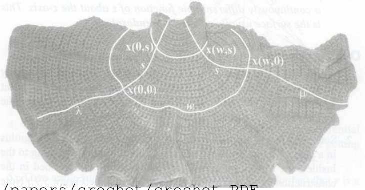

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$=$

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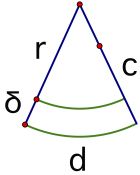

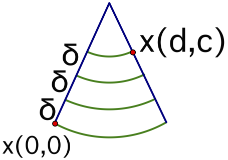

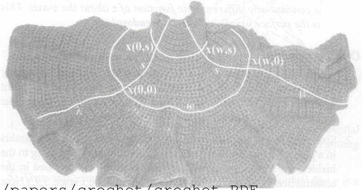

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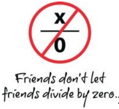
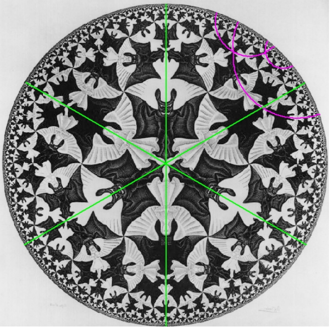


$= de^{-\frac{c}{r}}$

s-media-cache-ak0.pinimg.com/236x/c8/b0/cc/c8b0cc37c388075903aee52eda55e4c0.jpg

Knit Theory

Knit Theory & Artistic Representations

Dutch graphic artist M.C. Escher's Circle Limit IV: Heaven and Hell 1960

Latvian/US mathematician Daina Taimina *Crocheting Adventures with Hyperbolic Planes*

1st pic: Like a flat map of hyperbolic space

2nd pic: Annulus model is limit as $\delta \to 0$

Hyperbolic Approximations: More Models

http://www.math.tamu.edu/~frank.sottile/research/subject/stories/hyperbolic_football/

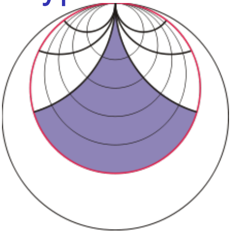
convoluted.jpg

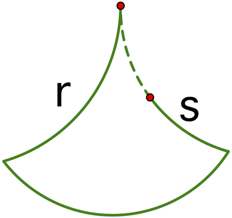
http://www.beriewede.com/

1st pic: Piecing together flat heptagons and hexagons

2nd pic: Annulus model is limit as $\delta \to 0$

Hyperbolic Gauss Curvature K and Angle Sum

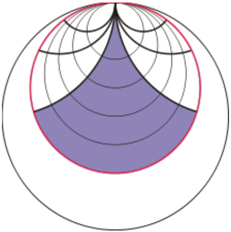




https://circulosmeos.files.wordpress.com/2014/10/220px-pseudoesfera_horodisco-svg.png

- s arc length along radial geodesic, so $|x_2| = 1$ and $|x_1| = e^{-\frac{s}{r}}$ and $g_{ij} = \begin{bmatrix} e^{-\frac{2s}{r}} & 0 \\ 0 & 1 \end{bmatrix}$
- Compute $K = -\frac{1}{2\sqrt{EG}}(\frac{\partial}{\partial s}(\frac{E_s}{\sqrt{EG}}) + \frac{\partial}{\partial w}(\frac{G_w}{\sqrt{EG}}))$, which is equivalent to the book's formula when $F = 0$.
- What does Gauss-Bonnet tell us about angle sum of a geodesic triangle?

SA Horocycle and Radial Geodesics d Apart



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$$lim_{b\to\infty} \int_0^d \int_0^b \sqrt{e^{\frac{-2s}{r}}} dsdw$$

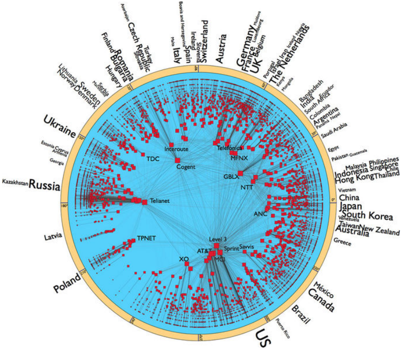
```
> simplify(limit(int(int(exp(-s/r),s=0..b),w=0..d),b=infinity))
```

$$\lim_{b \to \infty} \left(-r\, d \left(e^{-\frac{b}{r}} - 1 \right) \right)$$

Finite!

Applications

- Models of the internet to reduce the load on routers



Sustaining the Internet with hyperbolic mapping: Marian Boguna, Fragkiskos Papadopoulos & Dmitri Krioukov

- Building crystal structures to store more hydrogen or absorb more toxic metals

Modeling and Explaining Behavior

life.dpics.org

CC-BY-2.0 Margaret Wertheim

Quanta Magazine article on “What is the Geometry of the Universe” dated March 16, 2020:

> *Mathematicians like to say that it’s easy to get lost in hyperbolic space... your visual circle is growing exponentially, so your friend will soon appear to shrink to an exponentially small speck. If you haven’t tracked your friend’s route carefully, it will be nearly impossible to find your way to them later.*