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- Corollary: Consider $-\nabla\varphi$ and its relationship to $\frac{d^2\vec{X}}{dt^2}$
$\frac{d^2x}{dt^2} = -\frac{\partial\varphi}{\partial x}$, $\frac{d^2y}{dt^2} = -\frac{\partial\varphi}{\partial y}$, and $\frac{d^2z}{dt^2} = -\frac{\partial\varphi}{\partial z}$

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- Finite # point masses: φ is a sum in Laplace's eq
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- If $\Gamma^\lambda_{\mu\nu} = 0$ then $\frac{d^2x^\lambda}{ds^2} = 0$
- Field equations written in the Christoffel symbols:

$$\frac{\partial\Gamma^\lambda_{\mu\lambda}}{\partial x^\nu} - \frac{\partial\Gamma^\lambda_{\mu\nu}}{\partial x^\lambda} + \Gamma^\beta_{\mu\lambda}\Gamma^\lambda_{\nu\beta} - \Gamma^\beta_{\mu\nu}\Gamma^\lambda_{\beta\lambda} = 0$$

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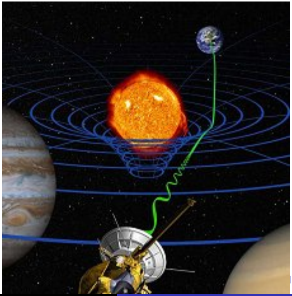
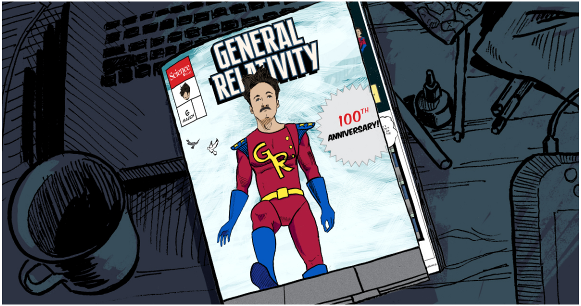
- Gravitational lensing and LIGO gravitational waves
- Precession of the orbit of Mercury



How many general-relativity theorists does it take to change a light bulb?



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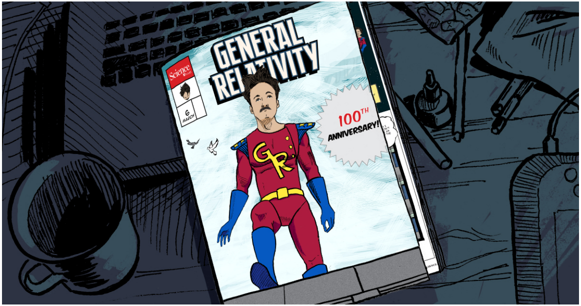
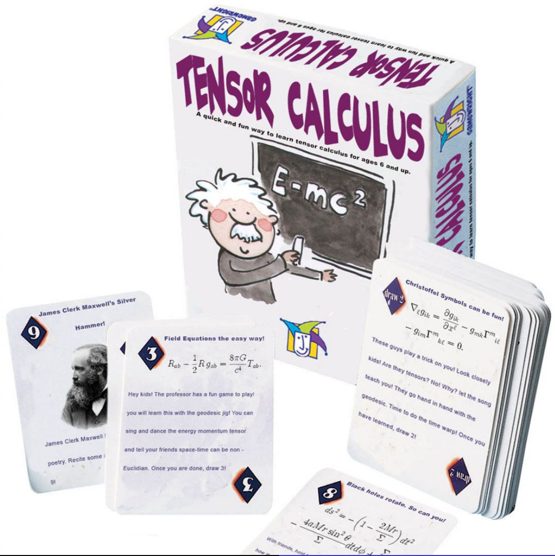


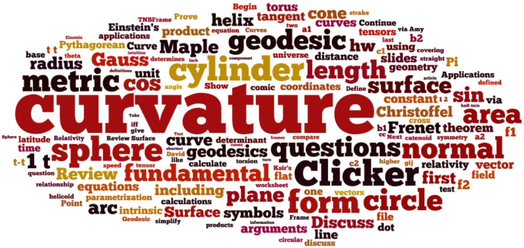
Two. One to hold the bulb and one to rotate space.

- Einstein's general relativity remains scientists' best understanding of gravity and a key to our understanding of the cosmos on the grandest scale [Theory of Everything?]



Course Overview

- Curves: torsion and curvature, piece together (DEs) to give us a nice curve. Frenet frame basis
- Surfaces: 1st and 2nd fundamental forms. 1st and Christoffel symbols intrinsic → Gauss curvature. E, F, G piece together to give us a nice smooth surface, or one with manageable singularities like the cone. l, m, n determine how the surface sits in space (if it does)
- SpaceTimes: g_{ij} and Γ^k_{ij} more terms → curvature tensors. Einstein's field equations: way to solve for "nice" solutions.



- Final Research Pres: review + extension + bib + peer review