Which topics from Calculus III and Analytic Geometry were a part of today's homework reading on p. 77–80 in 2.2 and 5.1 (209–215)?

a) dot product
b) cross product
c) derivatives
d) two of the above
e) all of a), b), and c)

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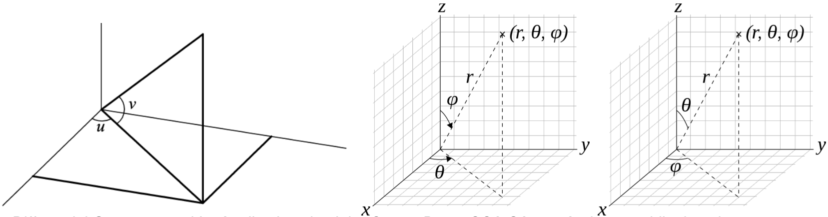
- E, F, G definitions and α'' dotted with various vectors
- U in various places, $U \times T$ in 5.1.6, geodesics on sphere are great circles in 5.1.8 $(\alpha' \times U)'$
- geodesic curvature depends only on the metric (E, F, G)
  geodesic has unit speed (we took $\alpha' = vT$)
  geodesics on sphere are great circles

Spherical Coordinates

geographical coordinates 2.1.9, 2.2.3, 5.1.8

$\mathbf{x}(u, v) = (r \cos u \cos v, r \sin u \cos v, r \sin v)$

- role of coordinates: hold one constant and explain what kind of curve the other gives, and then the reverse.
- $\vec{x}_u \times \vec{x}_v$ ever $\vec{0}$? What is U, E, F, and G?
- Interpret F—what does it tell us about the relationship between $\vec{x}_u$ and $\vec{x}_v$?



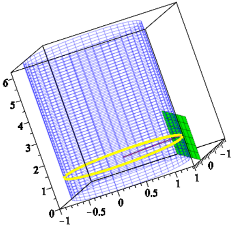
Differential Geometry and Its Applications by John Oprea, Dmcq CCA-SA 3.0, Andeggs public domain

spherical coordinates same u, different v

$\mathbf{x}(u, v) = (r \cos u \sin v, r \sin u \sin v, r \cos v)$

Maple File on Geodesic and Normal Curvatures

adapted from David Henderson



$\vec{\kappa}_\alpha$ pink dashed thickness 1
$\vec{\kappa}_n$ black solid thickness 2
$\vec{\kappa}_g$ tan dashdot style thickness 4

- The *unit normal* to the surface at a point is $U = \frac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$
- If $\vec{\kappa}_\alpha$ is the curvature vector for a curve $\alpha(t)$ on the surface then the *normal curvature* is the projection onto U:
$$\vec{\kappa}_n = (U \cdot \vec{\kappa}_\alpha)U$$
- The *geodesic curvature* is what is felt by the bug (in the tangent plane T_pM): $$\vec{\kappa}_g = \vec{\kappa}_\alpha - \vec{\kappa}_n$$

Commands for Maple File on Curvatures

```
g := (x,y) ->[cos(x)*cos(y), sin(x)*cos(y), sin(y)]
a1:=0: a2:=Pi: b1:=0: b2:=Pi:
c1 := 1: c2 := 3:
Point := 2:
f1:= (t) -> t:
f2:= (t) -> 1:
```

Implications of the Spherical Metric Form

$$(\frac{ds}{dt})^2 = E(\frac{du}{dt})^2 + 2F\frac{du}{dt}\frac{dv}{dt} + G(\frac{dv}{dt})^2$$

$$ds^2 = g_{11}(du^1)^2 + 2g_{12}du^1du^2 + g_{22}(du^2)^2 = \sum_{i,j} g_{ij}du^idu^j$$

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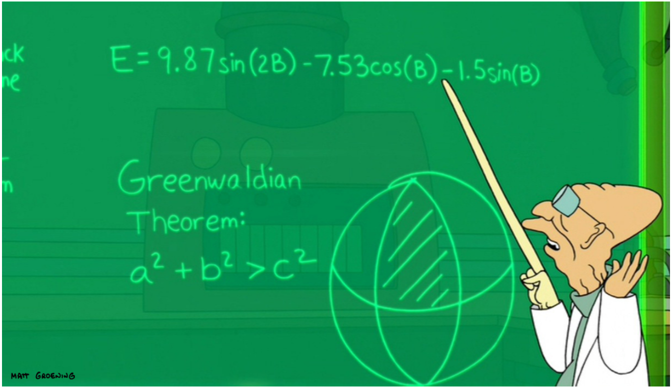

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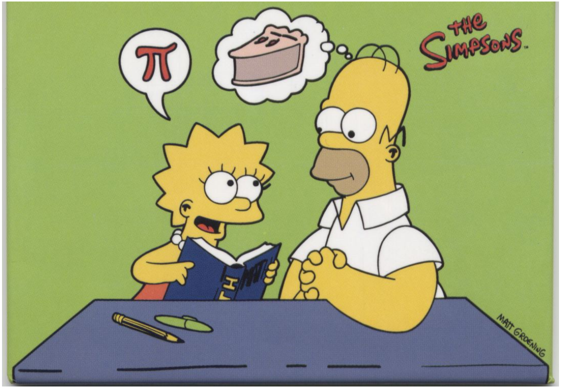
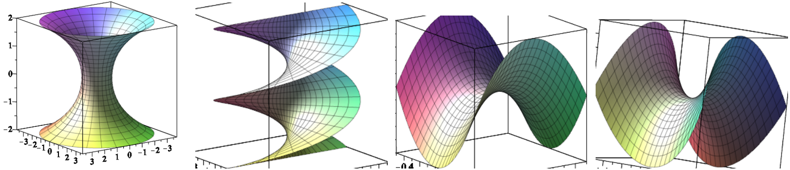
π on the Sphere? $\dfrac{\textit{circumference}}{\textit{diameter}}$

Definition of a circle? Estimate π_{sphere} for different circles.



Implications of the Metric Form



catenoid helicoid saddle Enneper's surface

catenoid $\mathbf{x}(u, v) = (\cosh u \cos v, \cosh u \sin v, u)$
helicoid $\mathbf{x}(u, v) = (\sinh u \cos v, \sinh u \sin v, v)$
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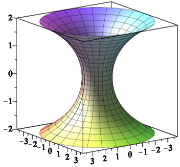
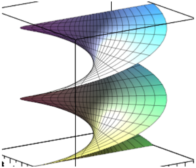
Enneper's surface $\mathbf{x}(u, v) = (u - \frac{u^3}{3} + uv^2, v - \frac{v^3}{3} + vu^2, u^2 - v^2)$

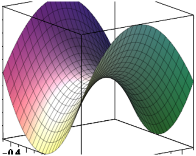
Implications of the Metric Form



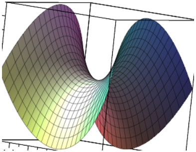
catenoid



helicoid



saddle



Enneper's surface

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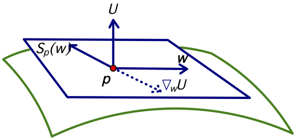
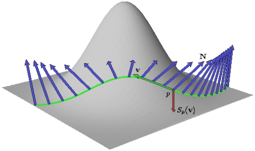
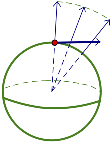
`http://virtualmathmuseum.org/Surface/helicoid-catenoid/helicoid-catenoid.mov`

2nd Fundamental Form $l = \vec{x}_{uu} \cdot U$, $m = \vec{x}_{uv} \cdot U$, $n = \vec{x}_{vv} \cdot U$







2nd picture: The Center of Population of the United States
http://www.ams.org/publicoutreach/feature-column/fcarc-population-center

- curve: κ, τ rate of change of unit vector fields T & B ($\therefore$ N).
- surface: U unit vector field. Whole plane of directions—rates of change of U are measured, not numerically, but by a linear operator called the shape operator, which captures the bending of a surface.

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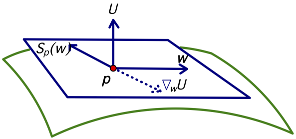

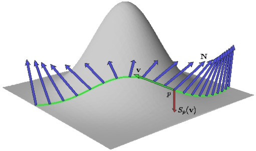

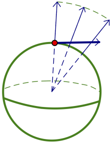

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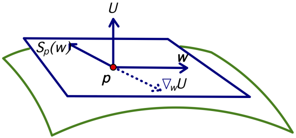

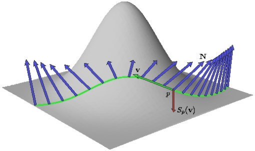

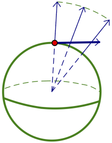

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- $S_p(\vec{w}) = -\nabla_{\vec{w}} U$
- $S(\vec{x}_u) \cdot \vec{x}_u = \vec{x}_{uu} \cdot U = l$, $S(\vec{x}_u) \cdot \vec{x}_v = \vec{x}_{uv} \cdot U = m$,
  $S(\vec{x}_v) \cdot \vec{x}_v = \vec{x}_{vv} \cdot U = n$
- eigenvalues of the shape operator: max and min normal curvature at p, called the principal curvatures κ_1 and κ_2

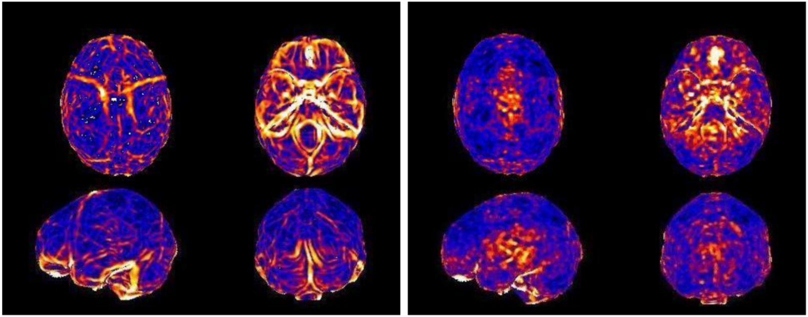

Fig. 3. Shape operator (left) and polynomial fit-derived (right) magnitude of curvature on the inner skull surface. Note that the shape operator is sensitive enough to assign high curvature to small structures, such as the vessel impressions on the inner skull surface. The polynomial-fit curvature image was processed with surface-constrained smoothing, to reduce noise, while the shape operator curvature did not require smoothing.

Avants, Brian and James Gee (2003) "The Shape Operator for Differential Analysis of Images," *Inf Process Med Imaging*. Jul 18:101–13.