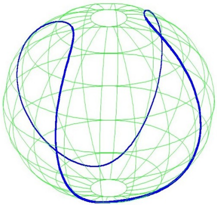
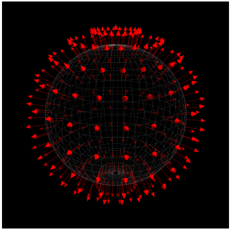
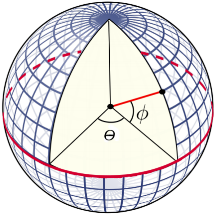
S^2: geodesic param by arc length must be a great circle







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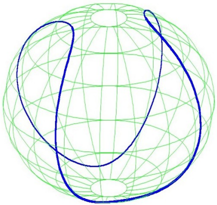

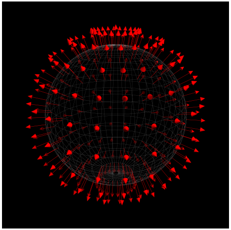

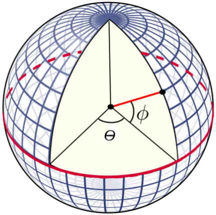

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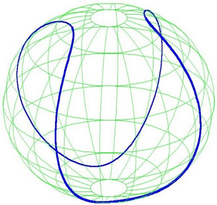

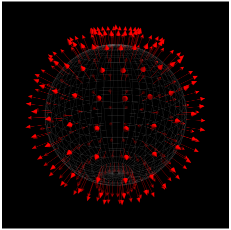

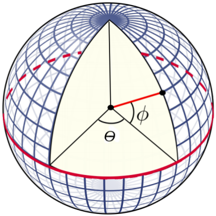

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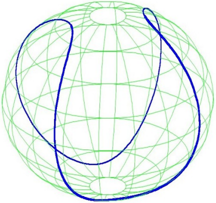

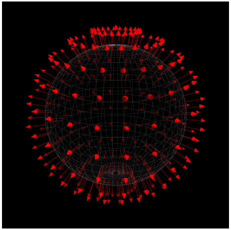

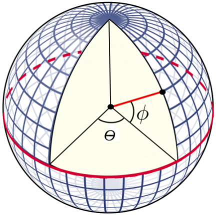

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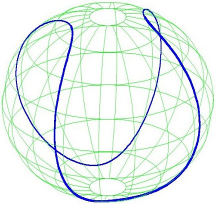

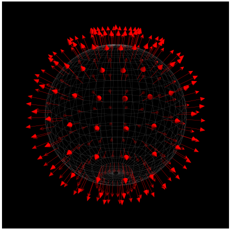

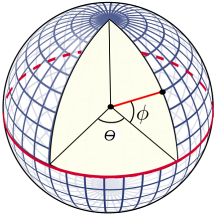

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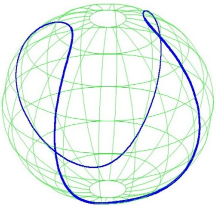

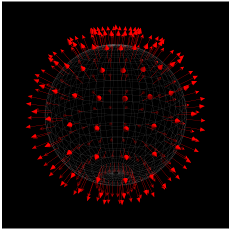

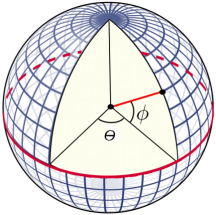

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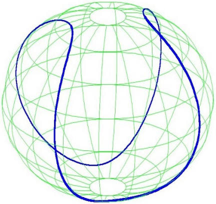

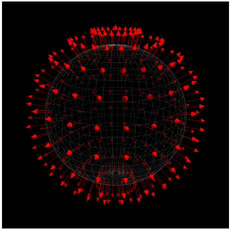

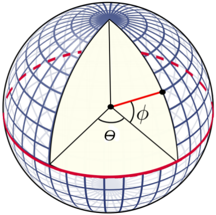

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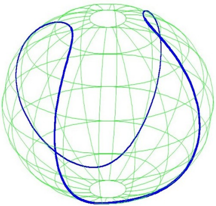

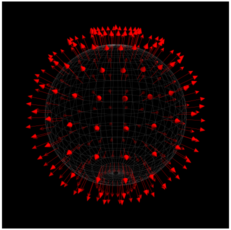

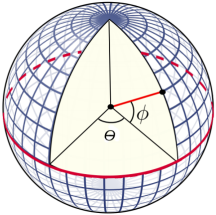

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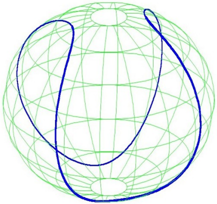

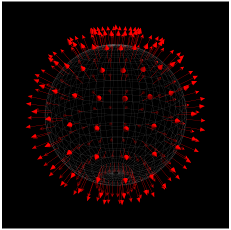

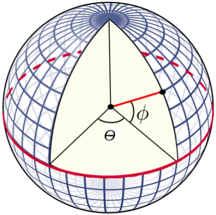

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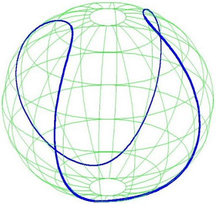

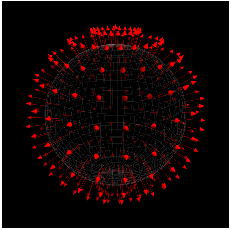

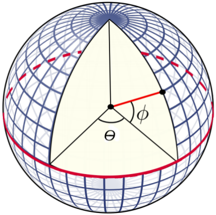

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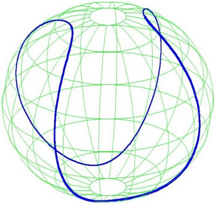

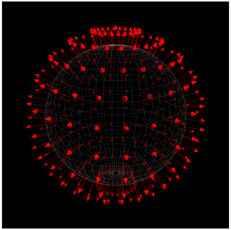

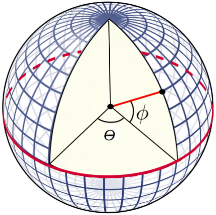

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  However $T \perp B$, so $|\kappa| = \frac{1}{R}$ & $\tau = 0$, ie circle radius R.