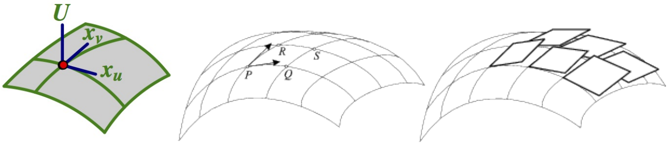
First Fundamental Form and Surface Area



- $\vec{x}_u \cdot \vec{x}_v = |\vec{x}_u||\vec{x}_v| \cos\theta$
- $|\vec{x}_u \times \vec{x}_v| = |\vec{x}_u||\vec{x}_v| \sin\theta$

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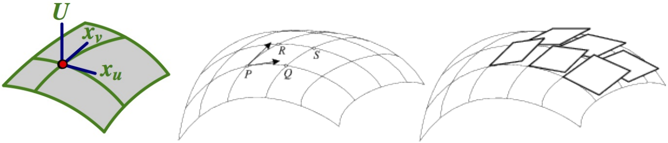

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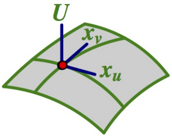

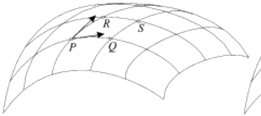

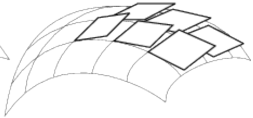

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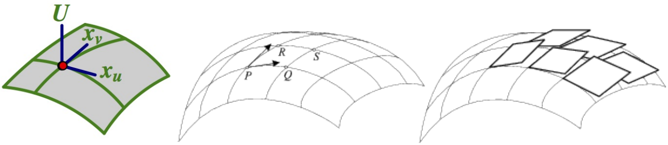

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- $\sin\theta = \sqrt{1 - \cos^2(\theta)} = \sqrt{1 - (\frac{\vec{x}_u \cdot \vec{x}_v}{|\vec{x}_u||\vec{x}_v|})^2}$

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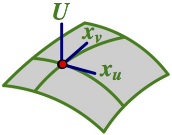

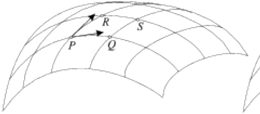

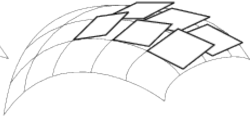

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$$\overset{\text{common denom}}{=} \sqrt{\frac{|\vec{x}_u|^2|\vec{x}_v|^2 - (\vec{x}_u\cdot\vec{x}_v)^2}{|\vec{x}_u|^2|\vec{x}_v|^2}}$$

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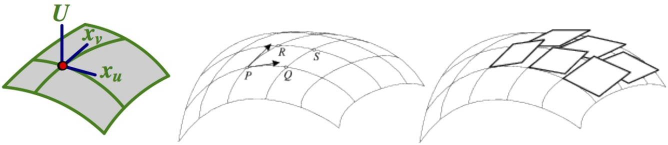

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$$\overset{\text{common denom}}{=} \sqrt{\frac{|\vec{x}_u|^2|\vec{x}_v|^2-(\vec{x}_u\cdot\vec{x}_v)^2}{|\vec{x}_u|^2|\vec{x}_v|^2}} = \sqrt{\frac{(\vec{x}_u\cdot\vec{x}_u)(\vec{x}_v\cdot\vec{x}_v)-(\vec{x}_u\cdot\vec{x}_v)^2}{|\vec{x}_u|^2|\vec{x}_v|^2}} = \sqrt{\frac{EG-F^2}{|\vec{x}_u|^2|\vec{x}_v|^2}}$$

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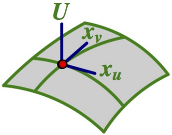

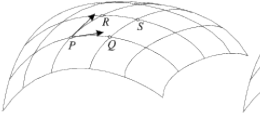

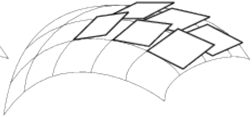

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So area $= |\vec{x}_u||\vec{x}_v| \sin\theta = \sqrt{EG-F^2}$

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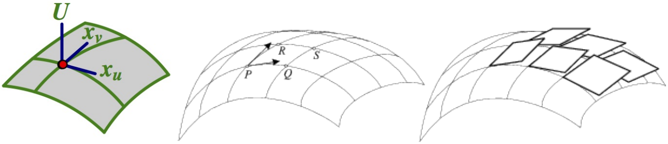

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  So area $= |\vec{x}_u||\vec{x}_v|\sin\theta = \sqrt{EG-F^2}$
- Let $\Delta u, \Delta v \to 0$ and add up over the entire surface: $\int\int\sqrt{EG-F^2}dudv$