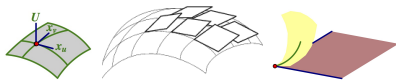
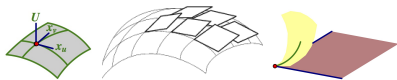


Proof of Surface Area with 1st Fundamental Form



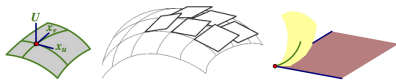
- $\vec{X}_U \cdot \vec{X}_V = |\vec{X}_U| |\vec{X}_V| \cos \theta$
- $|\vec{X}_U \times \vec{X}_V| = |\vec{X}_U| |\vec{X}_V| \sin \theta$

Proof of Surface Area with 1st Fundamental Form



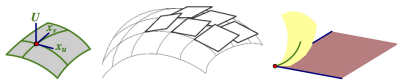
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Proof of Surface Area with 1st Fundamental Form



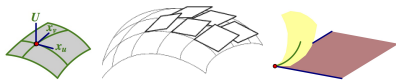
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Proof of Surface Area with 1st Fundamental Form



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Proof of Surface Area with 1st Fundamental Form

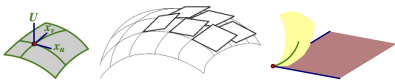


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$$\stackrel{\text{common denom}}{=} \sqrt{\frac{|\vec{x}_u|^2 |\vec{x}_v|^2 - (\vec{x}_u \cdot \vec{x}_v)^2}{|\vec{x}_u|^2 |\vec{x}_v|^2}}$$

Proof of Surface Area with 1st Fundamental Form

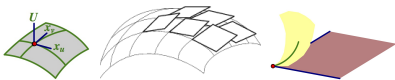


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Proof of Surface Area with 1st Fundamental Form



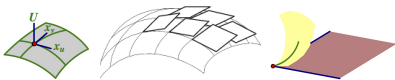
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So area = $|\vec{x}_u| |\vec{x}_v| \sin \theta = \sqrt{EG - F^2}$

Proof of Surface Area with 1st Fundamental Form



- $\vec{x}_u \cdot \vec{x}_v = |\vec{x}_u| |\vec{x}_v| \cos \theta$
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So area = $|\vec{x}_u| |\vec{x}_v| \sin \theta = \sqrt{EG - F^2}$

- Let $\Delta u, \Delta v \rightarrow 0$ and add up over the entire surface:

$$\iint \sqrt{EG - F^2} du dv$$

In our prior activities, for a round torus you should have calculated:

$$E = r^2 \quad F = 0 \quad \text{and} \quad G = (R + r \cos u)^2$$

To calculate the surface area of the round donut (mmmm frosting):

Peardoodles 

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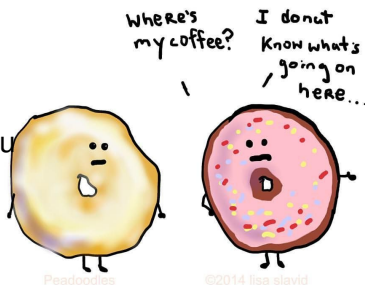
a) $\int_0^{2\pi} \int_0^{2\pi} r^2 (R + r \cos u)^2 \, dv \, du$

b) $\int_0^{2\pi} \int_0^{2\pi} r (R + r \cos u) \, dv \, du$

c) $4\pi^2 rR$

d) more than one holds

e) none of the above



glazed and confused



Strake Computations: I, SA, Shape Operator, II, K

$$\mathbf{x}(u, v) = (v \cos(u), v \sin(u), ku)$$

a) $\vec{x}_u =$

b) $\vec{x}_v =$

c) $g_{ij} =$

d) set up SA of one turn of the strake, when the inner radius is 1, the outer radius is 1.2, and the height of one turn is 10m.

e) $U =$

f) $S(\vec{x}_u) = -\nabla_{\vec{x}_u} U = -U_u =$

Then write it in the basis of $\vec{x}_u + \vec{x}_v$

g) $S(\vec{x}_v) = -\nabla_{\vec{x}_v} U = -U_v =$

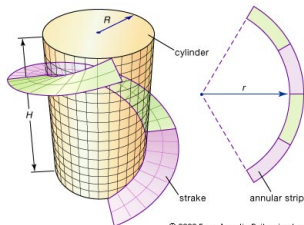
Then write it in the basis of $\vec{x}_u + \vec{x}_v$

h) $K =$

i) $l = S(\vec{x}_u) \cdot \vec{x}_u = \vec{x}_{uu} \cdot U =$

j) $m = S(\vec{x}_u) \cdot \vec{x}_v = \vec{x}_{uv} \cdot U =$

k) $n = S(\vec{x}_v) \cdot \vec{x}_v = \vec{x}_{vv} \cdot U =$



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Cylinder Computations: I, SA, Shape Operator, II, K

$$\mathbf{x}(\theta, z) = (r \cos \theta, r \sin \theta, z)$$

a) $\vec{x}_\theta =$

b) $\vec{x}_z =$

c) $g_{ij} =$

d) set up the double integral for surface area of a cylinder with height from $z = 0$ to h

e) $U =$

f) $S(\vec{x}_\theta) = -\nabla_{\vec{x}_\theta} U = -U_\theta =$

Then write it in the basis of $\vec{x}_\theta + \vec{x}_z$

g) $S(\vec{x}_z) = -\nabla_{\vec{x}_z} U = -U_z =$

Then write it in the basis of $\vec{x}_\theta + \vec{x}_z$

h) $K =$

i) $l = S(\vec{x}_\theta) \cdot \vec{x}_\theta = \vec{x}_{\theta\theta} \cdot U =$

j) $m = S(\vec{x}_\theta) \cdot \vec{x}_z = \vec{x}_{\theta z} \cdot U =$

k) $n = S(\vec{x}_z) \cdot \vec{x}_z = \vec{x}_{zz} \cdot U =$

SA of Surface Disk on Sphere and Cone

- SA of circular disk with center at N pole of intrinsic radius r in spherical coordinates on a sphere of radius R .

$$\lim_{\delta \rightarrow 0} \int_{v=\delta}^{v=\frac{r}{R}} \int_{u=0}^{u=2\pi} \sqrt{EG - F^2} dudv$$

- SA of circular disk of intrinsic radius r with the intrinsic center at the cone point.

$$\lim_{\delta \rightarrow 0} \int_{v=\delta}^{v=r} \int_{u=0}^{u=\alpha} \sqrt{EG - F^2} dudv$$

- firstfundformsa.mw