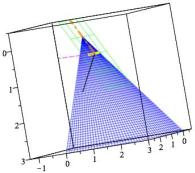
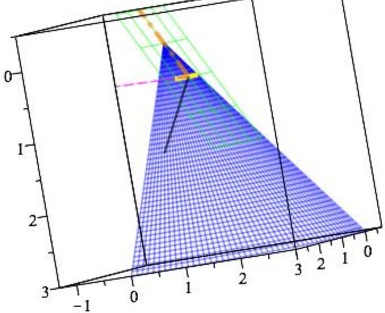
Surfaces in $\mathbb{R}^3$



- Let **x** (u, v) be an extrinsic parametrization in $\mathbb{R}^3$
  Cone: **x** (u, v) = $(v cos(u), v sin(u), v)$
  Cylinder: **x** (θ, z) = $(cos(\theta), sin(\theta), z)$
- $\vec{x}_u$ and $\vec{x}_v$ are tangent vectors
- The *unit normal* to the surface at a point is $U = \frac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$
- If $\vec{\kappa}_\alpha$ is the curvature vector for a curve $\alpha(t)$ on the surface then the *normal curvature* is the projection onto U:
  $$\vec{\kappa}_n = (U \cdot \vec{\kappa}_\alpha)U$$
- The *geodesic curvature* is what is felt by the bug (in the tangent plane T_pM): $$\vec{\kappa}_g = \vec{\kappa}_\alpha - \vec{\kappa}_n$$



$\vec{\kappa}_\alpha$ pink dashed thickness 1
$\vec{\kappa}_n$ black solid thickness 2
$\vec{\kappa}_g$ tan dashdot style thickness 4

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