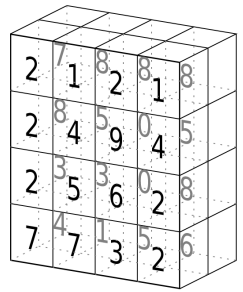
Tensors and the Metric Tensor g_{ij}

't'
'e'
'n'
's'
'o'
'r'

3	1	4	1
5	9	2	6
5	3	5	8
9	7	9	3
2	3	8	4
6	2	6	4



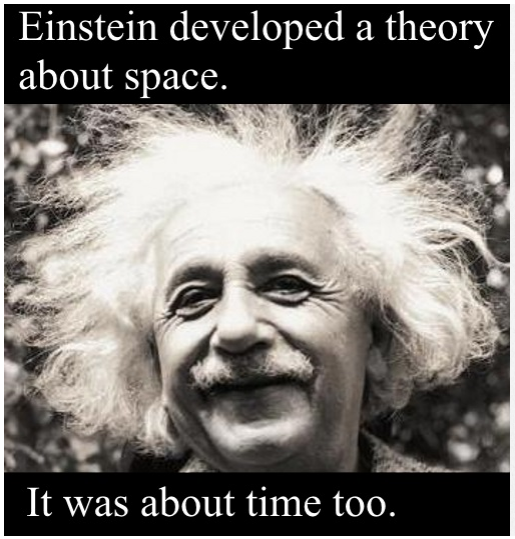
lists #s vectors stack of matrices

- algebraic combinations of vectors, matrices, vector spaces, algebras, modules or other structures
- often geometrically meaningful
- not all tensors are inherently linear maps

g_{ij} inner products of tangent vectors $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$

$$g^{ij} = g_{ij}^{-1} = \begin{bmatrix} E & F \\ F & G \end{bmatrix}^{-1} = \frac{1}{EG-F^2} \begin{bmatrix} G & -F \\ -F & E \end{bmatrix}$$

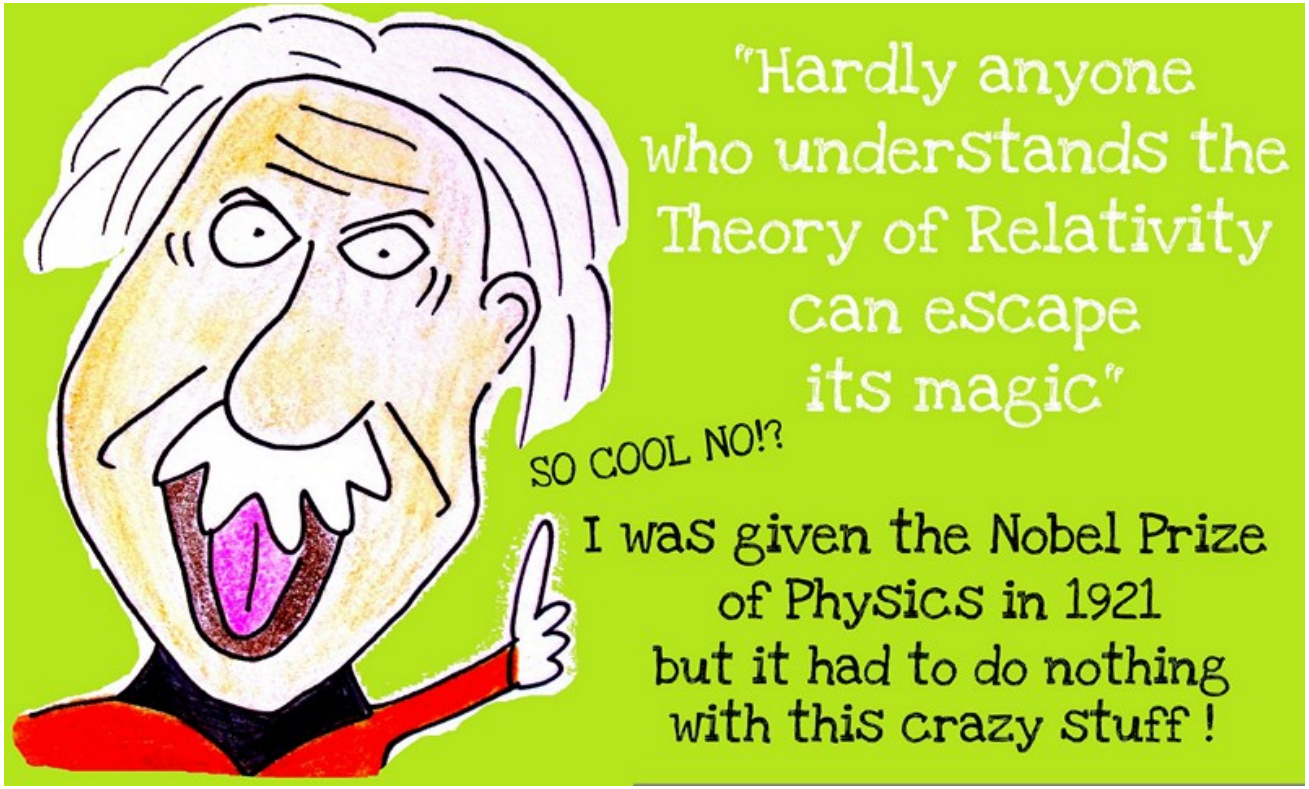
SpaceTime-Time: Special Relativity



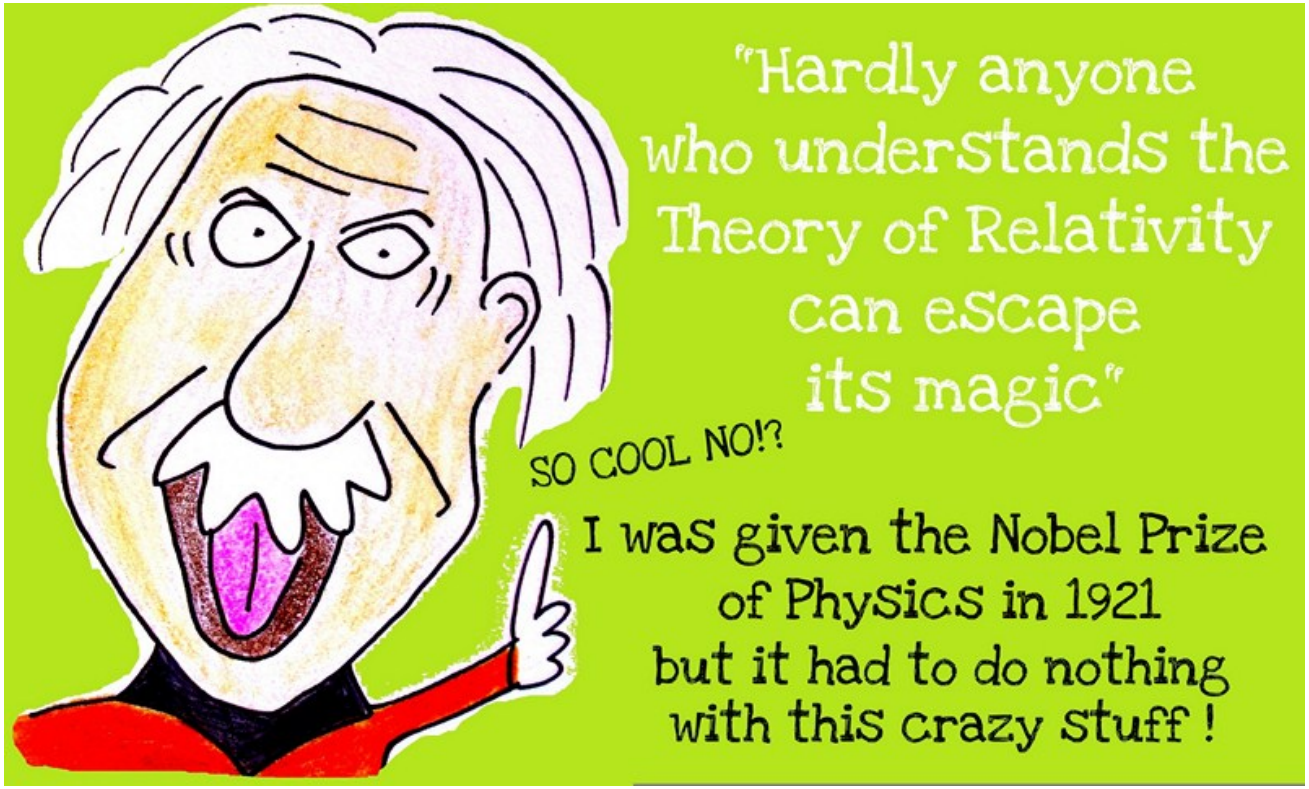
Albert Einstein special relativity (1905)
Hermann Minkowski 4D spacetime model (1908)
Surfaces: g_{ij} inner products of tangent vectors $w^T g_{ij} v$

$$\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

SpaceTime: Metric tensor is now 4x4 symmetric matrix acting as $w^T g_{ij} v$ on (t, x, y, z) vectors. Yardstick plus clock! We'll prove that free particles follow straight line geodesics.



Why should followers of special relativity not be taken seriously?



Why should followers of special relativity not be taken seriously?

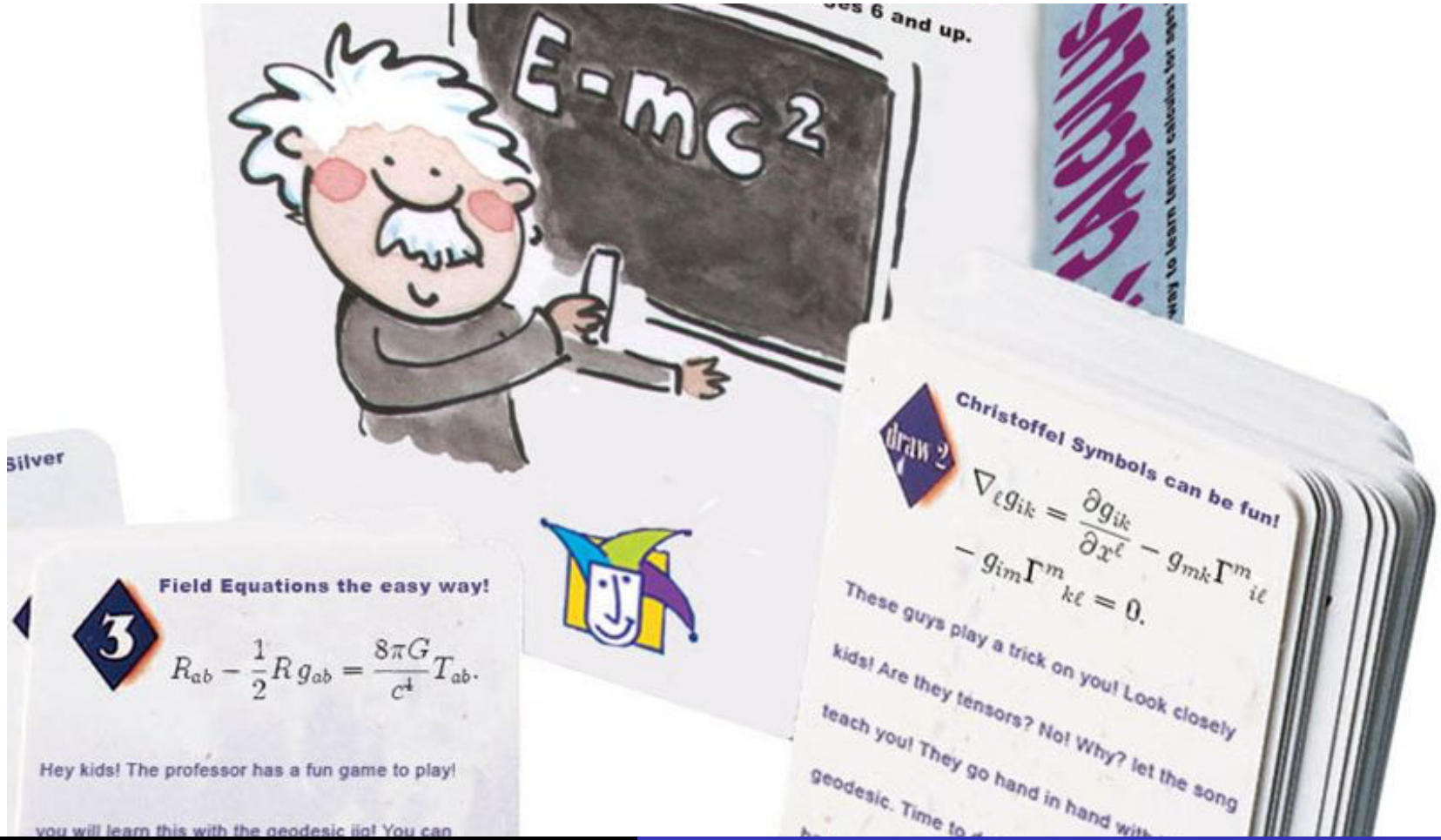
They fail to see the gravity of the situation!

SpaceTime-Time: Other SpaceTimes

4D Manifold, g_{ij}, curvature satisfy Einstein field equation
g_{ij} can be other 4x4 symmetric matrices acting as $w^T g_{ij} v$.
g_{ij} 1 eigenvalue > 0 and three eigenvalues < 0 at each point.
angle : $\cos\theta = \frac{w^T g_{ij} v}{|v||w|}$, spacetime interval: $|v| = \sqrt{v^T g_{ij} v}$

- change g_{ij} and change physical properties of SpaceTime
- astrophysicist begins with what we observe and tries to construct a metric that models it
- enter any metric, then use Einstein's field equation to read off the physical properties of the resulting universe

- Einstein's general relativity remains scientists' best understanding of gravity and a key to our understanding of the cosmos on the grandest scale
- Ricci curvature tensor depends on the metric via Christoffel symbols




TENSOR CALCULUS
A quick and fun way to learn tensor calculus for ages 6 and up.
E=mc²
James Clerk Maxwell's Silver Hammer!
9
Field Equations the easy way!
3
$R_{ab} - \frac{1}{2}R\,g_{ab} = \frac{8\pi G}{c^4}T_{ab}.$
Hey kids! The professor has a fun game to play! you will learn this with the geodesic jig! You can sing and dance the energy momentum tensor and tell your friends space-time can be non - Euclidian. Once you are done, draw 3!
Christoffel Symbols can be fun!
draw 2
$\nabla_\ell g_{ik} = \frac{\partial g_{ik}}{\partial x^\ell} - g_{mk}\Gamma^m_{i\ell} - g_{im}\Gamma^m_{k\ell} = 0.$
These guys play a trick on you! Look closely kids! Are they tensors? No! Why? let the song teach you! They go hand in hand with the geodesic. Time to do the time warp! Once you have learned, draw 2!
draw 2
8
Black holes rotate. So can you!
$ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4aMr\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma}{\Delta}dr^2$