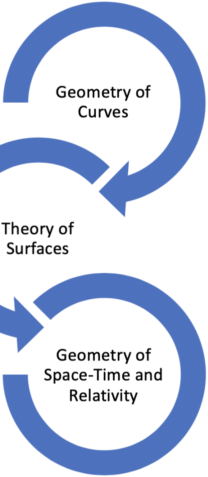
- review questions
- arc length of the tractrix $\alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$ by-hand and in Maple
- project 1



1. Which of the following could represent the line between the points (-3,2,5) and (1,-2,4)

a) $\begin{bmatrix} -3 \\ 2 \\ 5 \end{bmatrix} + t \begin{bmatrix} -4 \\ 4 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + t \begin{bmatrix} 4 \\ -4 \\ -1 \end{bmatrix}$

c) both of the above

d) none of the above

2. A line has

a) velocity is $\vec{0}$

b) acceleration is $\vec{0}$

c) acceleration is nonzero but constant

d) more than one of the above

3. In Euclidean space, the shortest distance path between $\vec{p}$ and $\vec{q}$ is the line $\vec{p} + t(\vec{q} - \vec{p})$ because:

a) No matter the geometry, we must head in the direction from $\vec{p}$ to $\vec{q}$ to achieve the shortest path

b) The length of the line = $\int_a^b \alpha'(t) \cdot \frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|} dt \leq \int_a^b |\alpha'(t)| |\frac{\vec{q}-\vec{p}}{|\vec{q}-\vec{p}|}| dt = \int_a^b |\alpha'(t)| dt$

c) both of the above

d) none of the above

4. The dot product of two vectors in 3-space, $\vec{v} \cdot \vec{w}$ is

a) $|\vec{v}||\vec{w}|cos(\theta)$ where θ is the angle between them

b) $|\vec{v}||\vec{w}|sin(\theta)$

c) $v_1 w_1 + v_2 w_2 + v_3 w_3$, where v_i is the ith entry of $\vec{v}$

d) more than one of the above

5. To calculate a tangent vector and the velocity vector

a) If the curve is parametrized by time then it is the same calculation

b) They are always equal

c) For tangent take the component derivatives with respect to the parameter in the parametrization, for velocity take the component derivatives with respect to time

d) more than one is true

6. Which of the following are true regarding arc length s?

a) $s = \int |\alpha'(t)| \mathrm{d}t$

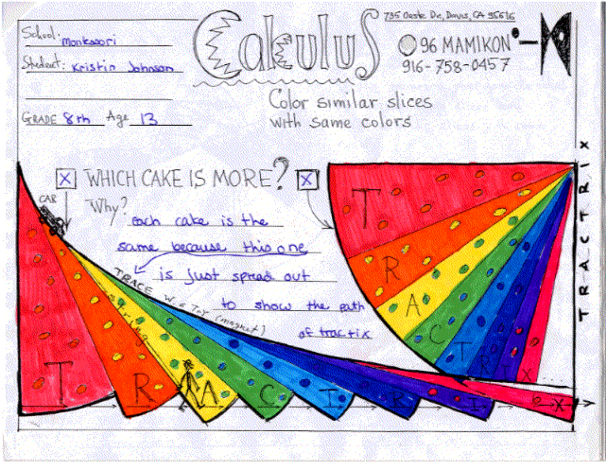
b) $s = \int \text{speed } \mathrm{d}t$

c) a closed form solution for s may not exist, even for regular curves

d) more than one is true

1.1 & 1.2: Tractrix and Arc Length

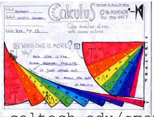
$\alpha(\theta) = (\cos(\theta) + \ln(\tan(\frac{\theta}{2})), \sin(\theta))$



https://www.its.caltech.edu/~mamikon/Mont.html

1.1 & 1.2: Tractrix and Arc Length

$$\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}} |\alpha'(\theta)| d\theta \text{ where } \alpha(\theta) = (\cos(\theta) + \ln(\tan(\tfrac{\theta}{2})), \sin(\theta))$$



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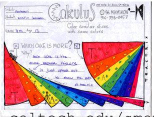

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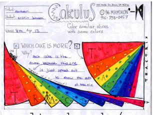

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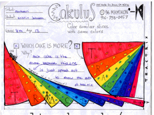

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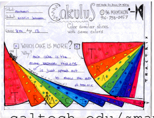

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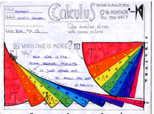

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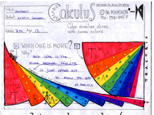

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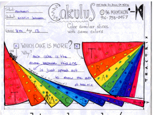

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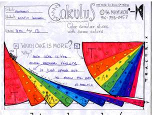

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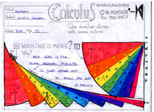

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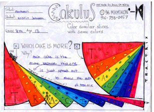

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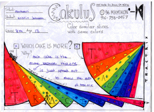

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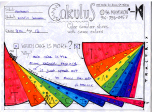

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pythagorean-magnitude-t-shirt-24.jpg

Project 1 Introduction

- List your preferred first name(s)
- Image
- Handwrite or professionally typeset general formulas for the following entities as a review in equations and/or words. Assume that you have a curve parametrized in time. Do NOT do any calculations for your specific curve here...
- Explain in your own words what each of the items from the last question generally means physically and/or geometrically, connected to the language of our class...
- Adapt Maple file diffgeomproj1.mw—at the bottom, I have parameterizations for your curve...

Project 1 Introduction

- Historical mathematicians, physicists, engineers, or others who are related to your curve
- Search for additional information on one person
  - their contributions or connections to your curve
  - the title of their publication that included content related to your curve or a year or a range of years they worked on your curve, if possible. Or if not, then their year of birth and, if applicable, death, would provide their working years
  - what country they worked in
  - something you found interesting about the person
- MathSciNet for a journal article related to your curve

- summarize the significance of your curve in historical and current research including (if possible) real-life applications or connections as well as the earliest year you can find related to your curve.
- summarize the physically interesting features of your curve.
- proper credit
- collate Maple and other work into one PDF
- elevator pitch presentation about your curve

"I've got an elevator pitch, an escalator pitch, and, just to be safe, a stairway pitch."