Template: Maximum 4-Page Review & Annotated References

By: Dr. Sarah

1 Section Header

Include the relevant definitions, mathematical symbols and notation, theorems, important proofs, and examples in order to review the major concepts from class and homework.

2 Metric Balls

The Euclidean distance formula on $\mathbb{R}^2$, is given by

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}.$$

2.1 Definition (Metric Ball). *For each x in X, define $B(x, \epsilon)$, the open ball of radius ϵ centered at x, as $\{y \mid d(x, y) < \epsilon\}$. Define the collection of open balls associated with the metric d as*

$$\mathcal{B} = \{B(x, \epsilon) | x \in X, \epsilon > 0\}$$

2.2 Theorem. $\mathcal{B} = \{B(x, \epsilon) | x \in \mathbb{R}^2, \epsilon > 0\}$ *is a basis for* $\mathbb{R}^2$.

Proof of the Theorem. Since each point $x \in \mathbb{R}^2$ is contained in the basis element $B(x, 1)$, the first condition of being a basis is satisfied. Now we need to check that if x is a point in the intersection of two basis elements, there is a basis element containing x and contained in the intersection. Let $x \in B(y, r_1) \cap B(z, r_2)$. Notice $\exists \epsilon_1, \epsilon_2 > 0$ such that $B(x, \epsilon_1) \subset B(y, r_1)$ and $B(x, \epsilon_2) \subset B(x, r_2)$. Let $\epsilon = \min\{\epsilon_1, \epsilon_2\}$. Then

$$B(x, \epsilon) \subset B(x, \epsilon_1) \cap B(x, \epsilon_2) \subset B(y, r_1) \cap B(z, r_2),$$

completing the proof that $\mathcal{B}$ is a basis. $\square$

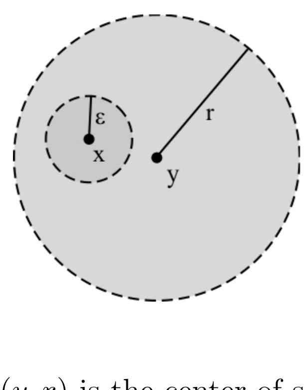

Figure 1: Every point x in $B(y, r)$ is the center of some ball contained in $B(y, r)$.

3 Section Header

3.1 Sample Subsection Header

3.1 Example (Rotation Matrix).

$$A = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

represents a counterclockwise rotation by θ in the $x-y$ plane with the z coordinate fixed.

4 Comparing Topologies

4.1 Theorem. *Prove that the standard topology $(\mathbb{R}^2, d)$ equals the square topology $(\mathbb{R}^2, \rho)$.*

Proof of the Theorem. To show that $(\mathbb{R}^2, d) = (\mathbb{R}^2, \rho)$, we will show that $(\mathbb{R}^2, d) \subseteq (\mathbb{R}^2, \rho)$ and $(\mathbb{R}^2, d) \supseteq (\mathbb{R}^2, \rho)$.

$\supseteq$ Let $U \in (\mathbb{R}^2, \rho)$ and let $x \in U$. We will show that U is open in $(\mathbb{R}^2, d)$, ie that $\exists\epsilon > 0$ so that $B_d(x, \epsilon) \subseteq U$. By the definition of open in $(\mathbb{R}^2, \rho)$ we know $\exists\epsilon > 0$ so that $B_\rho(x, \epsilon) \subseteq U$. Take the same ϵ and let $y \in B_d(x, \epsilon)$. We will show that $y \in U$. By definition of metric balls, we know that $d(x, y) \leq \epsilon$. In addition, by the 4th problem on the Metric Space Exercises (Mendelson p. 34 # 3), we have already proved that $\rho(x, y) \leq d(x, y)$, and so $\rho(x, y) < \epsilon$. Hence $y \in B_\rho(x, \epsilon) \subseteq U$, as desired.

$\subseteq$ Let $U \in (\mathbb{R}^2, d)$ and let $x \in U$. We will show that U is open in $(\mathbb{R}^2, \rho)$. By the definition of open in $(\mathbb{R}^2, d)$ we know $\exists\epsilon > 0$ so that $B_d(x, \epsilon) \subseteq U$. Take $\frac{\epsilon}{\sqrt{2}}$. Let $y \in B_\rho(x, \frac{\epsilon}{\sqrt{2}})$. To show that $y \in U$, using the same Mendelson problem as above, $d(x, y) \leq \sqrt{2}\rho(x, y) \leq \sqrt{2}\frac{\epsilon}{\sqrt{2}} = \epsilon$. Hence $y \in B_d(x, \epsilon) \subseteq U$ so U is open in $(\mathbb{R}^2, \rho)$.

$\square$

4.1 Citations

Here are in text citations for the bibliography [1, 2, 3, 4, 5] which should be placed where appropriate, like [1].

4.2 Mathematical Commands

Here are some sample mathematics commands

The term costs below illustrate 1-year term costs for three different wood products

Assume 1 ton of production (tons CO_2 potential), initial cost $b = \$50$ per ton CO_2, inflation rate in the cost of CO_2, $r = 2\%$ and a long term continuous discount rate, $\delta = 5\%$.

Year	Waste, bark, fuel	Pulpwood	Fencing
1	\$4.69	\$19.08	$\$4.0x10^{-8}$
2	\$5.81	\$14.95	$\$3.6x10^{-6}$
3	\$5.68	\$8.28	$\$4.6x10^{-5}$
4	\$5.25	\$4.23	$\$2.5x10^{-4}$
5	\$4.72	\$2.08	$\$9.0x10^{-4}$
PV Total Cost	\$23.74	\$46.15	$\$1.0x10^{-3}$
5-year Term	\$23.74	\$46.15	$\$1.0x10^{-3}$

The expected cost over the initial term period is given by

$$\overline{C}_{0:n} = E[b_T \overline{Z}_n] = \int_0^n b_t e^{-\delta t} P(t) dt \tag{4.2}$$

The cost over the next n-years can be found,

$$\overline{C}_{x:n} = \int_0^n b_{x,t} e^{-\delta_x t} f_T(t) dt$$

$$\overline{C} = \sum_{k=0}^{\infty} e^{-\delta k} \int_k^{\infty} P(t) dt \, \overline{C}_{k:1}$$

$e^{-\delta k}$ is the discount factor discounting back to time $t = 0$, $\int_k^{\infty} P(t)dt$ is the expected percentage of stock left undecayed at time $t = k$, and $\overline{C}_{k:1}$ is the one year term cost per ton of CO_2 emitted during the time interval $t = k$ to $t = k + 1$ (year $k + 1$) from the stock that has been exposed to the decay hazard for $k-$years and survived.

Command	**Type Setting**
`$A \cap B$`	$A \cap B$
`$ \bigcap \limits _{n=1} ^{\infty}$`	$\bigcap\limits_{n=1}^{\infty}$
`$A\cup B$`	$A \cup B$
`$\bigcup \limits _ \alpha$`	$\bigcup\limits_{\alpha}$
`$\subseteq$`	$\subseteq$
`$\subsetneq$`	$\subsetneq$
`$\supsetneq$`	$\supsetneq$
`$\emptyset$`	$\emptyset$
`$\mathbbR$`	$\mathbb{R}$
`$\in$`	$\in$
`$\notin$`	$\notin$
`$\in$`	$\in$
`$\Rightarrow$`	$\Rightarrow$
`$\Leftarrow$`	$\Leftarrow$
`$\Leftrightarrow$`	$\Leftrightarrow$
`$\exists$`	$\exists$
`$\forall$`	$\forall$
`$B_d(x,\epsilon )$`	$B_d(x, \epsilon)$
`$U _1,...,U _n \in X $`	$U_1, ..., U_n \in X$
`$T _\alpa \in T $`	$T_\alpha \in T$
`$\textbf{to make bold letters}$`	**to make bold letters**
Textinmathmode `\mbox {text in an mbox}`	*Insidemathmode*text in an mbox

$$\overline{Z}_n = \begin{cases} e^{-\delta T} & , 0 \leq T \leq n \\ 0 & , T > n \end{cases}$$

5 Annotated Bibliography

Use many different types of sources, including scholarly references and library sources. Submit a separate annotated bibliography of all of the sources you used in your project, with annotations explaining what content in the reference relates to your topic, how you used each reference, where the pictures came from, etc. Use as many pages as you need for the bibliography and annotations.

References

[1] Cullen, Charles. *Matrices and Linear Transformations, 2nd edition.* Reading, MA: Addison-Wesley Publishing Company, 1972

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[2] Kastrup, Hans A. *On the Advancements of Conformal Transformations and their Associated Symmetries in Geometry and Theoretical Physics.* Annalen der Physik (v. 17/9-10, 2008)

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[3] Kleiner, Israel. *A History of Abstract Algebra.* Boston: Birkhauser, 2007

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[4] O'Connor, John and Edmund Robertson. *MacTutor History of Mathematics Archive: Search Results for Transformations.* Available online: `http://www-history.mcs.st-andrews.ac.uk/Search/historysearch.cgi?SUGGESTION=transformations&CONTEXT=1` (accessed February 2011)

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[5] Rosenfeld, Boris Abramovich. *A History of Non-Euclidean Geometry: Evolution of the Concept of a Geometric Space.* New York: Springer, 1988

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