Rotation Matrix - Geometry

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} =$$

Rotation Matrix - Geometry

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

Rotation Matrix - Geometry

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow$$

Rotation Matrix - Geometry

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Rotation Matrix - Geometry

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Using the unit circle, we see that the output is on the $y = \tan(\theta)x$ line, ie the same length vector but θ degrees counterclockwise.

$$\begin{bmatrix} 2 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 2\cos(\theta) \\ 2\sin(\theta) \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow$$

Rotation Matrix - Geometry

$$\begin{bmatrix}\cos(\theta) & -\sin(\theta)\\ \sin(\theta) & \cos(\theta)\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}\cos(\theta)x - \sin(\theta)y\\ \sin(\theta)x - \cos(\theta)y\end{bmatrix}$$

$$\begin{bmatrix}1\\ 0\end{bmatrix} \rightarrow \begin{bmatrix}\cos(\theta)\\ \sin(\theta)\end{bmatrix}$$

Using the unit circle, we see that the output is on the $y = \tan(\theta)x$ line, ie the same length vector but θ degrees counterclockwise.

$$\begin{bmatrix}2\\ 0\end{bmatrix} \rightarrow \begin{bmatrix}2\cos(\theta)\\ 2\sin(\theta)\end{bmatrix}$$

$$\begin{bmatrix}0\\ 1\end{bmatrix} \rightarrow \begin{bmatrix}-\sin(\theta)\\ \cos(\theta)\end{bmatrix}$$

Rotation Matrix - Algebra

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

Rotation Matrix - Algebra

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} r\cos(\phi) \\ r\sin(\phi) \end{bmatrix} = \begin{bmatrix} r\cos(\theta)\cos(\phi) - r\sin(\theta)\sin(\phi) \\ r\sin(\theta)\cos(\phi) - r\cos(\theta)\sin(\phi) \end{bmatrix}$$

Rotation Matrix - Algebra

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos(\theta)x - \sin(\theta)y \\ \sin(\theta)x - \cos(\theta)y \end{bmatrix}$$

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} r\cos(\phi) \\ r\sin(\phi) \end{bmatrix} = \begin{bmatrix} r\cos(\theta)\cos(\phi) - r\sin(\theta)\sin(\phi) \\ r\sin(\theta)\cos(\phi) - r\cos(\theta)\sin(\phi) \end{bmatrix}$$

$$= \begin{bmatrix} r\cos(\theta + \phi) \\ r\sin(\theta + \phi) \end{bmatrix}$$

Transformations of $\mathbb{R}^2$

Rotation Matrix: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

Projection Matrix: $\begin{bmatrix} \cos(\theta)^2 & \cos(\theta)\sin(\theta) \\ \cos(\theta)\sin(\theta) & \sin(\theta)^2 \end{bmatrix}$

Dilation Matrix: $\begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}$

Horizontal Shear: $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

Reflection Matrix: $\begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix}$

- What important transformations are missing?

Translation Matrix: $\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x + h \\ y + k \end{bmatrix}$

$$\text{Translation Matrix: } \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \end{bmatrix}$$

$$\text{Translation Matrix: } \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$$

$$\text{Translation Matrix: } \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \end{bmatrix}$$

$$\text{Translation Matrix: } \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+h \\ y+k \\ 1 \end{bmatrix}$$

$$\text{Rotation Matrix: } \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate a Figure about the point $\begin{bmatrix} 4 \\ 9 \end{bmatrix}$:

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & -9 \\ 0 & 0 & 1 \end{bmatrix}$$