Alcubierre metric or warp drive metric

$$ds^2 = -dt^2 + (dx - v_s(t)f(r_s)dt)^2 + dy^2 + dz^2$$

t is time
x, y, z are rectangular coordinates in space
$x_s(t)$ is motion of a spaceship along the $x - axis$
$v_s(t) = \frac{dx_s(t)}{dt}$
r_s is the distance from the spaceship position
$\sqrt{(x - x_s(t))^2 + y^2 + z^2}$
$f(r_s) = \frac{\tanh(\sigma(r_s+R))-\tanh(\sigma(r_s-R))}{2\tanh(\sigma R)}$
$R > 0$ radius of the spherical warp bubble
$\sigma > 0$ is the bubble thickness

Citation: Varieschi, Gabriele U. and Zily Burstein (2013). "Conformal Gravity and the Alcubierre Warp Drive Metric" *ISRN Astronomy and Astrophysics*

anti-de Sitter metric

$$ds^2 = \cosh^2 r dt^2 - a^2(dr^2 + \sinh^2 r\sigma)$$

t time
r radial
$a = \sqrt{\frac{3}{\lambda}}$ where λ is a cosmological constant
$\sigma = d\theta^2 + \sin^2\theta d\phi^2$ is the standard metric on the 2-sphere S^2, with θ, ϕ angular.

Citation: Juan Antonio Valiente Kroon (2016). *Conformal Methods in General Relativity*, Cambridge University Press

de Sitter metric for special relativity

$$ds^2 = dt^2 + a^2 \cosh^2(\frac{t}{a}) \quad \bar{h}$$

t time
$a = \sqrt{\frac{3}{\lambda}}$ where λ is a cosmological constant
$\bar{h}$ is a metric of the 3-sphere S^3

Citation: Juan Antonio Valiente Kroon (2016). *Conformal Methods in General Relativity*, Cambridge University Press

Eddington-Finkelstein coordinates

From the Schwarzschild metric, change the time coordinate t to u and v, which are defined by
$u = t - r - 2m \log |r - 2m|$
$v = t + r + 2m \log |r - 2m|$
m is a mass parameter
r is radial

$$ds^2 = \frac{1}{2}(1 - \frac{2m}{r})(dudv + dvdu) - r^2\sigma$$

$\sigma = d\theta^2 + \sin^2\theta d\phi^2$ is the standard metric on the 2-sphere S^2, with θ, ϕ angular.

Citation: Juan Antonio Valiente Kroon (2016). *Conformal Methods in General Relativity*, Cambridge University Press

Friedmann-Lemaıtre-Robertson-Walker (FLRW) metric

$$ds^2 = -dt^2 + a^2(t)d\Omega_3^2$$

$d\Omega_3^2 = d\chi^2 + f_K^2(\chi)(d\theta^2 + \sin^2\theta d\theta^2)$ is the metric of a 3-sphere, a 3-plane, or a 3-hyperboloid
$f_K = \{\sin, I, \sinh\}$ corresponding to $K = \{-1, 0, 1\}$
t is cosmic time
$a(t)$ has the dimension of a length and is a scale factor.

Citation: Deruelle, Nathalie and Jean-Philippe Uzan (2018). *Relativity in Modern Physics*, Oxford Graduate Texts

Gödel metric

$$ds^2 = a^2(dx_0^2 - dx_1^2 + dx_2^2 \frac{1}{2} e^{2x_1} - dx_3^2 + 2e^{x_1} dx_0 dx_2)$$

$a > 0$ is a constant

Citation: Patrick Marquet (2021). “The Exact Gödel Metric” *Progress in Physics*

Gullstrand-Painlevé or Painlevé-Gullstrand coordinates

$$ds^2 = -(1 - \frac{2m}{r})d\tau^2 + 2\sqrt{\frac{2m}{r}}drd\tau + d\overrightarrow{x}^2$$

where $t \to \tau(t, r)$ via

$\tau = t + \int dr \frac{\sqrt{\frac{2m}{r}}}{1-\frac{2m}{r}} = t + 2m(2\sqrt{\frac{r}{2m}} + \ln(\frac{|\sqrt{\frac{r}{2m}}-1|}{\sqrt{\frac{r}{2m}}+1}))$

t time

m mass

r radial

$\overrightarrow{x}$ spherical

Citation: Deruelle, Nathalie and Jean-Philippe Uzan (2018). *Relativity in Modern Physics*, Oxford Graduate Texts

Kerr metric

$$ds^2 = -(1 - \frac{2mr}{\rho^2})dT^2 + \frac{\rho^2}{r^2+a^2}(1 + \frac{2mr}{r^2+a^2})dr^2 + \rho^2 d\theta^2$$
$$+ (r^2 + a^2 + \frac{2mra^2 \sin^2\theta}{\rho^2})\sin^2\theta d\phi^2$$
$$+ 4mr(\frac{1}{r^2+a^2}dTdr - \frac{a\sin^2\theta}{\rho^2}dTd\phi - \frac{a\sin^2\theta}{r^2+a^2}drd\phi)$$

T time
r radial
θ, ϕ angular
m mass
a angular momentum per unit mass
$\rho^2 = r^2 + a^2\cos^2\theta$

Citation: Deruelle, Nathalie and Jean-Philippe Uzan (2018). *Relativity in Modern Physics*, Oxford Graduate Texts

Kerr-Newman rotating charged black hole metric

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - a\sin^2\theta d\phi)^2 + \frac{\sin^2\theta}{\rho^2}((r^2+a^2)d\phi - adt)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

t time
r radial
θ, ϕ angular
m mass
a angular momentum per unit mass
$\Delta = r^2 - 2mr + a^2 + q^2$, with q electric charge
$\rho^2 = r^2 + a^2\cos^2\theta$

Citation: Deruelle, Nathalie and Jean-Philippe Uzan (2018). *Relativity in Modern Physics*, Oxford Graduate Texts

Kruskal-Szekeres coordinates

$$ds^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2) - 32m^3 r^{-1} e^{-\frac{r}{2m}} dudv$$

r radial
θ, ϕ angular
t time
m mass
$u = -\sqrt{\frac{r}{2m} - 1}\, e^{\frac{r}{4m}} e^{-\frac{t}{2m}}$
$v = -\sqrt{\frac{r}{2m} - 1}\, e^{\frac{r}{4m}} e^{\frac{t}{2m}}$

Citation: Hans Stephani et al. (2003). *Exact Solutions of Einstein's Field Equations*. Cambridge University Press

Lemaître coordinates

$$ds^2 = -d\tau^2 + \frac{1}{\frac{r}{2m}} d\rho^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\tau = t + \int dr \frac{\sqrt{\frac{2m}{r}}}{1-\frac{2m}{r}} = t + 2m(2\sqrt{\frac{r}{2m}} + \ln(\frac{|\sqrt{\frac{r}{2m}}-1|}{\sqrt{\frac{r}{2m}}+1}))$$

$$\rho = t + \int dr \frac{1}{\sqrt{\frac{2m}{r}}(1-\frac{2m}{r})} = t + 2m(\sqrt{\frac{r}{2m}}(2 + \frac{r}{3m}) + \ln(\frac{|\sqrt{\frac{r}{2m}}-1|}{\sqrt{\frac{r}{2m}}+1}))$$

t time

r radial

m mass

θ, ϕ angular

$r(\rho, \tau) = 2m(\frac{3}{4m}(\rho - \tau)^{\frac{2}{3}})$

Citation: Deruelle, Nathalie and Jean-Philippe Uzan (2018). *Relativity in Modern Physics*, Oxford Graduate Texts

Minkowski metric/space

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

t is time
x, y, z are rectangular coordinates in space

Reissner-Nordström metric

$$ds^2 = -(1 - \frac{2m}{r} + \frac{q^2}{r^2})dt^2 + \frac{dr^2}{1 - \frac{2m}{r} + \frac{q^2}{r^2}} + r^2 d\Omega_2^2$$

t time
θ, ϕ angular
r radial
$d\Omega_2^2 = d\theta^2 + \sin^2\theta d\phi^2$
$m = GM$ mass of the black hole
$q = \sqrt{G}M$ electric charge of the black hole

Citation: Deruelle, Nathalie and Jean-Philippe Uzan (2018). *Relativity in Modern Physics*, Oxford Graduate Texts

Rindler coordinates

$$ds^2 = -\tilde{z}^2 d\tilde{t}^2 + dx^2 + dy^2 + d\tilde{z}^2$$

t time
x, y, z rectangular
$\tilde{z} = \sqrt{z^2 - t^2}$
$\tilde{t} = arctanh \frac{t}{z}$

Citation: Griffiths, Jerry and Jiřý Podolský (2010). *Exact Space-Times in Einstein's General Relativity*, Cambridge University Press

Schwarzschild metric

$$ds^2 = (1 - \frac{2m}{r})dt^2 - (1 - \frac{2m}{r})^{-1}dr^2 - r^2\sigma$$

m is a mass parameter
r is radial
t is time
$\sigma = d\theta^2 + \sin^2\theta d\phi^2$ is on the 2-sphere S^2, with θ, ϕ angular.

Citation: Juan Antonio Valiente Kroon (2016). *Conformal Methods in General Relativity*, Cambridge University Press

Taub-NUT metric

$$ds^2 = -f(r)(dt + 4l \sin^2 \frac{\theta}{2} d\phi)^2 + \frac{1}{f(r)} dr^2 + (r^2 + l^2)(d\theta^2 + \sin^2 \theta d\phi^2)$$

t time
r radial
θ, ϕ angular
$m \geq 0$ constant
l constant, known as the NUT parameter
$f(r) = \frac{r^2 - 2mr - l^2}{r^2 + l^2}$

Citation: Griffiths, Jerry and Jiřý Podolský (2010). *Exact Space-Times in Einstein's General Relativity*, Cambridge University Press

Weyl-Lewis-Papapetrou coordinates

$$ds^2 = e^{2\psi}(dt - \omega d\phi)^2 - e^{2(\gamma-\psi)}(d\rho^2 + dz^2) - \rho^2 e^{-2\psi} d\phi^2$$

t time
ϕ angular
ρ radial
z rectangular
ω angular velocity, a function of ρ
γ, ψ functions of ρ

Citation: M. Sharif (2007). “Energy-Momentum Distribution of the Weyl-Lewis-Papapetrou and the Levi-Civita Metrics” *Brazilian Journal of Physics*

wormhole metric

$$ds^2 = (-1 + \frac{r_0}{r} - \frac{\epsilon}{r^2})dt^2 + \frac{1}{1 - \frac{r_0}{r}}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

t is time
r is radial
θ, ϕ are angular
ϵ is electric charge
r_0 is smallest radius of throat

Citation: Amy Ksir's Maple file