MAT 5530: Selected Topics in Differential Geometry

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Contents

1.1 Catalog Description and Course Objectives

Catalog Description: MAT 5530-5549 - Selected Topics (1-4). When Offered: On Demand

We consider the differential geometry of curves, surfaces, and spacetime, including theoretical and computational components, intrinsic and extrinsic viewpoints, and numerous applications. Prerequisite: linear algebra, multivariable calculus, differential equations, analysis, analytical physics. Graduate students who are enrolled in selected topics will attend the undergraduate course 4140. They will complete that work as well as extra graduate problems. For example, for the final project graduate students will have an additional component to research the literature (mathematics and/or physics and/or cs journals) and discuss some recent work, and if possible an open problem, that relates to the topic. Graduate students satisfy the undergraduate learning goals for the course as well as some additional goals:

Undergraduate Learning Goals

- To develop geometric skills and 3-D spatial visualization skills.
- To develop a greater appreciation for connections between various disciplines of mathematics, including geometry, linear algebra, analysis, and differential equations, along with an introduction to these subjects as they apply to differential geometry.
- To understand the importance of differential geometry in various scientific fields, including physics.
- To practice critical and creative thinking and to communicate effectively with your peers

(Additional) Graduate Learning Goals

- To build off of existing coursework in analysis, differential equations and analytical physics
- Research, review and interpret literature related to topics in differential geometry

1.2 Required Resources

- *Differential Geometry and Its Applications*, John Oprea, MAA Press: An Imprint of the American Mathematical Society, ISBN: 978-1-4704-5050-2, 2007.

- reliable access to technology, software, and high speed connectivity: The work you're going to do outside of class you'll be accessing though your internet connection with a device, so it will be really important to have access to a dependable high-speed internet connection and a good computer that can run everything we'll need. The software is free, including Maple—any student with a valid Appstate email address can access Maple on their own computer or on campus computers. For optional office hours, we'll use Zoom videoconference software with breakout rooms. Flexible browsers that will play common media formats from various sources such as from webpages, Google Drive, YouTube, and ASULearn, including interactive videos, are also something we'll use. ASULearn components work best from scrolling through the activities themselves on a computer—the Moodle mobile app does not always show everything as designed, both for visibility and for due dates.

- scanning handwritten work: the course is designed so that you'll collate responses, by-hand work, and Maple work for projects into a full size multipage PDF for submission in ASULearn. Electronically, you can append a Maple PDF you create to the end of a handwritten or professionally typeset PDF, like Preview on a Mac or PDFsam on a PC. Alternatively, if you are printing physically to paper, you can print your Maple work and then append it to the end of the handwritten work and then scan it. If you have a phone or tablet,

apps like Adobe Scan or CamScanner can work well to scan work to one full size multipage PDF. You can also use many printers or photo copiers to scan to PDFs—the school library lists that as an option and they can help: `https://library.appstate.edu/services-search/print-zone-tech-help`.

1.3 Assignment Types, Grades, and Policies

- **Effective ASULearn Engagement** 30%
  ASULearn checkmarks may be ones where you can manually mark the activity as completed or are earned for a good faith effort when you access an activity or receive a proficient grade by a deadline  . While some items have strict deadlines, there is still flexibility built in and multiple pathways for success—videos have multiple chances to succeed. Attempt readings and videos for completion and take video notes by the listed date when possible as the material builds on itself. To accommodate for emergencies, the lowest 3 checkmark assignments are dropped. Activities may include  web pages, PDFs, files, interactive videos, surveys, forums, turn-ins, begin in-class assessment guide, select a topic, and peer review. Examples are readings from the book as well as the following short readings: *Curves* and *Surfaces*, both by Doğan Çömez, myself and Jill Thomley, *How Flies Fly: Kappatau Space Curves* by Rudy Rucker, *How to Create Your Own Universe in Three Easy Steps* by Lawrence Brenton, and *Relativity* by David Brink.

- **Projects** 45%
  There are 4 projects over the course of the semester, which include research, investigations, and presentations to your classmates. The fourth project is during our assigned time at finals: "an instructor may NOT change the date or time of an examination without permission of the departmental chair and dean... Permission is granted only in case of emergency." Your work must be turned in on or before the due date at the beginning of class*. To accommodate issues that may arise, there is a revision opportunity for one of the first three projects.

- **In-class Assessments** 20%
  There are two timed in-class assessments designed to help you solidify and make connections. To encourage these as a learning experience, accommodate for emergencies, and help solidify your knowledge, there is a revision opportunity on one of them. Otherwise, no make ups allowed*.

- **Effective Class Engagement** 5%
  ASU designates that "Face-to face component is not a lecture but provides time for discussion, demonstrations, problem-solving, and higher-level thinking and collaborative activities. Class time is used to apply course content in ways that can only be accomplished when everyone is together in the same place." `https://cae.appstate.edu/teaching-learning/course-delivery-options`. Effective class engagement is essential to our course integrity and attendance is required at ALL such classes, with the exception of legitimate or excused absences.

  If you expect to miss more than 10% of classes due to university sponsored activities or other reasons then I advise you to drop the course. Any student who wants to obtain an "excused absence" for less than 10% of classes must meet certain responsibilities, including contacting me on the ASULearn forum in advance when possible, providing official documentation, and making up any possible work in advance.

  If the university cancels classes or changes them, continue progressing on the work due on ASULearn and check ASULearn for any updated info, which may include plans for the missed class such as Zoom meetings or individual activities.

  It will be useful if you bring a computer, tablet, or phone to classes that can access webpages—if not, you can still participate in other ways.

  ASU prepares students to employ various modes of communication that can help communities reach consensus or respectful disagreement: successful communicators interact effectively with people of both similar and different experiences and values and in this class you will practice oral and written communication during class by interacting with your peers and me. Regardless of gender, political party, race, religion, sexuality, or more this class is to be a welcoming environment, and so I want you to be sensitive and respectful to each

other in upcoming discussions. Keep it a safe place to express meaningful ideas and opinions. Actively listen to others and encourage everyone to participate. Part of the welcoming environment is to keep an open mind as you engage in our class activities, explore consensus and employ collective thinking across barriers. Maintain a professional tone, show respect and courtesy, and make your contributions matter. Performing activities that detract from the welcoming environment or distract your neighbors or me will result in a lowered grade. Asking or answering related and thought-provoking questions, coming up with creative ways of thinking about the material, and explaining the material to others are some examples of positive class engagement that will increase your grade.

* Accommodations in the determination of your final grade will be made for extenuating circumstances that are officially documented to prevent you from completing work early/on time.

The grading scale is: $A \geq 93; 90 \leq A- < 93; 87 \leq B+ < 90$...

1.4 Tentative Calendar

This class has two of our three credit hours as in-person meetings. You'll have daily work between our classes but you have the flexibility to work ahead to meet the deadlines if that is better for your schedule: plan to spend 3.5–5 hours between classes, on average, as per the University-wide Statement on Student Engagement with Courses—while we don't meet synchronously for the third hour, its time in and out of class are a part of this computation.

Details for assignments and in-class activities and any changes are on
`https://www.appstate.edu/~greenwaldsj/class/4140/s22.html`
and ASULearn.

	Class Monday	Between Classes (by just before 1pm Wed.)	Class Wednesday	Between Classes (by just before 1pm Monday)
1/10–1/12	review 2130 obtain rental book from bookstore	-class intro interactive video -read "Curves" -read 1.1 pp. 1–7 -lines and Maple intro interactive video -download or access Maple	curvature osculating circle parabola and line	-read 1.1 pp. 8–14 -tractrix interactive video -add ASULearn profile pic -Zoom update & profile pic -get to know posting -read the syllabus
1/19	State Holiday		arc length and speed comparing and contrasting curves	-read 1.2 pp. 14–17 -s, T and physical attributes interactive video -practice submitting PDF
1/24–1/26	s, T, velocity, speed, acceleration, jerk helix computations	-read 1.3 pp. 17–19 -TNB 1 interactive video -choice of curve for Project 1	TNB curve of Archytas cycloid and spiral	-read "How Flies Fly" -read 1.3 pp. 19–20 -TNB 2 interactive video
1/31–2/2	TNB spherical epitrochoid matching activity	-read 1.3 21–25 -curvature and torsion implications 1 interactive video -re-engage matching	curvature and torsion Darboux vector fundamental theorem of space curves	-Project 1: research, investigate and present a curve
2/7–2/9	Project 1 presentations	-read 1.5 pp. 34–35 -curvature and torsion implications 2 interactive video -begin assessment guide	curvature and torsion helix and strake	-prepare for in-class curves assessment -complete any open items
2/14–2/16	in-class curves assessment	-surfaces, geodesics and coverings interactive video -read pp. 247–250	covering geodesics cone	-read pp. 67–68, 77–82, 209 -coordinates and geodesic curvature interactive video

2/21–2/23	geodesics sphere	-read pp. 70–76, 212 -speed of a geodesic interactive video	geodesics round donut	-read "Surfaces" -first fundamental form interactive video -choose surface for Project 2
2/28–3/2	geodesics metric form flat and round donuts	-read pp. 83–87 -shape operator interactive video	shape operator	-read pp. 88–91, 91–96, 107–108, 111–114, 123–124 -II and Gauss's Theorema Egregium interactive video
3/14–3/16	π-day Gauss and mean curvature	-read p. 164 -surface area interactive video	surface area matching activity	-read pp. 275–277, 289–292 -Gauss Bonnet video -re-engage matching
3/21–3/23	Gauss Bonnet	-Project 2: research, investigate and present a surface	Project 2 presentations	-read pp. 226–235 -surfaces not embedded interactive video -begin assessment guide
3/28–3/30	surfaces not in $\mathbb{R}^3$ Klein bottles hyperbolic	- prepare for in-class surfaces assessment -complete any open items	in-class surfaces assessment	-read pp. 397–416 -geodesic equations, tensors and spacetime interactive video
4/4–4/6	spacetime and metric forms	-read "How to Create Your Own Universe in Three Easy Steps" -Minkowski spacetime and Christoffel computations interactive video -choose metric for Project 3	Christoffel symbols and geodesics metric form research suggestions	-read pp. 416–430 -wormhole metric, curvatures and relativity interactive video
4/11–4/13	curvatures recording final project suggestions	-begin final project	work on project 3 or final project	-Project 3: research, investigate and present a metric form
4/18–4/20	Project 3 presentations	-general relativity and the field equations interactive video -read "Relativity"	relativity concluding activities	-course survey -course evaluations
4/25–4/27	work on final project or optional revisions	-complete any open items	share final project idea or title	- final project video
4/29	turn in video presentation on ASULearn by the beginning of our 2pm assigned time during the assigned time, conduct video project peer review and self-evaluation (optional) revisions on one in-class assessment, one of the first three projects			

1.5 Course Communication, Office Hours, Where to Get Help, and Additional Policies

- Office Hours and ASULearn Forum: My office hours are on Zoom Sunday, Tuesday, and Thursday 7–7:45pm and Monday and Wednesday 8:15–9am via the link in the need help forum on ASULearn.

  I encourage you to talk to me often and you can ask me questions. I can also set up individual breakout rooms for private conversation with me. You can come in to Zoom to work on future assignments, working along and asking questions as they arise, or you can ask me questions about items you already turned in, or ask me to go over a concept. Keep me informed about any problems and I also want to hear when things are going well! You do not need to make an appointment to meet with me in Zoom, just drop by and I am happy to help!

  If you can't make it to Zoom, you can contact me on the Need Help Forum on ASULearn, which I'll try to answer at least once a day, including the weekends. Except for extreme emergencies, all private written communication must be handled through your private forum, with just you and I, on ASULearn rather than e-mail. I prefer that you use the optional Zoom office hours as it is easier to discuss material in person.

- During class, use of technology is allowed only when it is related to our class. Photos or video or audio recordings may not be taken in class without prior permission. There is no eating or drinking allowed in the classroom but you may step out if you need to hydrate or similar. Many activities are designed to be completed during class and you are responsible for all material and announcements, whether you are present or not. You are also responsible for announcements made on the web pages, so check them often.

- Academic integrity is a fundamental part of the course, which includes meeting deadlines, regular communication, and giving proper reference where it is due. These are essential to course integrity. Feel free to talk to me or each other if you are stuck, but when writing up work, be sure to give acknowledgment where it is due. Submitting someone else's work as your own (PLAGIARISM) is a serious violation of the University's Academic Integrity Code, which defines: "Plagiarism includes, but is not limited to, borrowing, downloading, cutting and pasting, and paraphrasing without acknowledgement, including from online sources, or allowing an individual's academic work to be submitted as another's work."

- I believe that you have the capability to succeed in this course. In this course, two of our three credit hours are in-person meetings. You'll have daily work between our classes but you have the flexibility to work ahead to meet the deadlines if that is better for your schedule: plan to spend 3.5–5 hours between classes, on average, as per the University-wide Statement on Student Engagement with Courses (while we don't meet for the third hour, its time in and out of class are a part of this computation).

  If you find that you are spending fewer hours, you can probably improve your understanding and grade by studying more. If you are (on average) spending more hours than these guidelines suggest, you may be working inefficiently; in that case, you should come see me.

- We adhere to University-wide syllabus and policy statements:
  `https://academicaffairs.appstate.edu/resources/syllabi-policy-and-statement-information`
  and University policies like on `https://policy.appstate.edu/Policy_Manual`

- I want you to be informed about your choices regarding what you tell me about certain types of sensitive information. In situations where students disclose experiencing an act of interpersonal violence to their instructor, faculty are required to report that to the campus Title IX Coordinator, who then reaches out to the student by email offering support services. I care about you and want you to get the resources you need. I'm happy to talk with you if you decide you want that, but please be aware that if instead you'd like to explore options with someone who can keep your information totally confidential, I highly recommend the Counseling Center at 828-262-3180. They offer walk-in hours as well as after-hours coverage: `http://counseling.appstate.edu`.

- Appalachian Cares is a place to find updates about matters of student health and safety `http://appcares.appstate.edu/`.

- The ASU Success in Online Courses website catalogs many of the university's academic and community supports to help you continue your academic endeavors. The website includes technology support for online and hybrid courses, study skills support, and connections to support services `https://studentlearningcenter.appstate.edu/about/success-online-courses`.

- Belk Library offers a wide array of research services including access to their digital media studio (with assistance available), video and audio recording rooms, Research Advisory Program sessions in which you meet individually to work with a librarian, Library guides, study spaces, and online workshops, like `http://library.appstate.edu/gethelp/rap`

- The writing center can help you with drafts of your reflections `https://writingcenter.appstate.edu/`

- Disco Student Learning Center offers academic resources that will complement and enhance classroom experiences by helping students become acquainted with their studies and learn how to learn effectively. For example, you can meet with a study skills specialist to discuss your goals and develop strategies and study plans to meet those goals or arrange for tutoring. "D.D.," or Dauphin Disco, was one of the three people who, in 1899, founded Watauga Academy – known as App State today. The word disco is Latin for learn. `https://studentlearningcenter.appstate.edu/students/tutoring-support-services`

- Prerequisite and Co-requisite Material:

  It will be helpful to review to make sure you understand concepts from Calculus With Analytic Geometry III, the prerequisite, and know how to do a computational example as we use them in class. This includes acceleration and velocity, arc length, cross product, curvature, cylindrical and spherical coordinates, derivative of a function of one variable whose range is in $\mathbb{R}^3$, i.e. $(x(t), y(t), z(t))$, directional derivative, dot product, equation of a line in 3-space, equation of a plane, fundamental theorem of calculus, gradient, magnitude, norm or length of a vector, multivariable chain rule, normal vector, parameterizations of curves and surfaces, partial derivatives of a multivariable function i.e. $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$for $f(x, y)$, speed, surface area, tangential and normal components of a vector like acceleration, tangent line, tangent plane, tangent vector, and volume. It would be helpful to pay special attention as these concepts as they come up in Linear Algebra, the co-requisite, and in differential geometry, or review them if you have already taken the class: addition of matrices, a_{ij} matrix notation, basis of a space, determinant of a matrix, dimension of a space, eigenvalue λ from $A\vec{x} = \lambda\vec{x}$, linear combination of vectors, multiplication of matrices, including $A\vec{v}$ and AB, inverse of a 2x2 matrix, span of vectors, symmetric matrix $A^T = A$, transpose of a matrix $A^T = (a_{ji})$ for $A = (a_{ij})$.

- The Conference Board of the Mathematical Sciences (CBMS) published a statement titled "Active Learning in Post-Secondary Mathematics Education" about the importance of "classroom practices that engage students in activities, such as reading, writing, discussion, or problem solving, that promote higher-order thinking" and our classroom is modeled after that. The purpose of engagement is to learn and practice course content and learning goals, and develop critical thinking and problem-solving skills. Making mistakes is integral to the learning process—the key is to try to continue to engage rather than give up—and this course is to be an environment in which you ask questions and offer good guesses. It is on purpose that there are problems that don't look exactly like what we did previously in order to provide you with rich settings to explore in order to learn deeply.

  I do not expect you to be able to solve all the issues immediately. Instead, I want to see what you can do on your own. Out in the real world, this is important, since no matter what job you have, you will be expected to seek out information and answers to new topics you have not seen before. This may feel uncomfortable and frustrating. I understand this and want to help you through the process. It helps to remember that there are no mathematical dead-ends! Each time we get stuck, it teaches us something about the problem we are working on, and leads us to a deeper understanding of the mathematics. In the real world though, you are not expected to face your work alone, and in this class is similar.

  Asking questions, and explaining things to others, in or out of class, is one of the best ways to improve your understanding of the material. I am always happy to help and will try to give you hints and direction to

help you understand the material. At times though, to encourage the exploration process, I may direct you to rethink a problem and to come back to discuss it with me again afterwards. This occurs when I believe that the struggle to understand is imperative for your deep understanding of the material.

1.6 Instructor Bio

I am a full Professor of Mathematics, and I am also an affiliate of Gender, Women's and Sexuality Studies (GWS) and the Math and Science Education Center (MSEC), investigating the connections between mathematics and society. My PhD in Riemannian geometry is from the University of Pennsylvania. I am married to the bassist Joel Landsberg. In our spare time, we like to travel, hike and conduct genealogy research. In addition to my own personal genealogy, I like to give back to the broader community, and in this context, I am also affiliated with ASU's center for Judaic, Holocaust and Peace Studies. Some of what I like about mathematics is also what I enjoy about genealogy—the sense of exploration, discovery and aha moments that come with lots of patience and effort.