Helix Applications of Curvature and Torsion

- $T(t) = \frac{\alpha'(t)}{|\alpha'(t)|}$
- $\vec{\kappa} = \frac{T'(t)}{|\alpha'(t)|}$
- $\kappa = |\vec{\kappa}|$
- $N(t) = \frac{\vec{\kappa}}{|\vec{\kappa}|}$
- $B(t) = T \times N$
- τ: compute $\frac{B'(t)}{|\alpha'(t)|}$ & compare it to N (they are multiples of each other) to find $-\tau$ and then τ
- $T'(s) = \kappa N$
  $N'(s) = -\kappa T + \tau B$
  $B'(s) = -\tau N$
  $$\begin{bmatrix} T'(s) \\ N'(s) \\ B'(s) \end{bmatrix} = \begin{bmatrix} 0 & \kappa & 0 \\ -\kappa & 0 & \tau \\ 0 & -\tau & 0 \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix}$$

Prove that $\alpha(s)$ with $\frac{\tau}{\kappa}$ constant ($\kappa \neq 0$) is a cylindrical helix, a helix where $T \cdot \vec{u}$ is constant.

http://previews.123rf.com/images/limbi007/limbi0071302/limbi007130200034/17726502-Orange-cartoon-characters-runs-on-the-green-helix--Stock-Photo-orange-spiral.jpg

Assume $\kappa \neq 0$ and $\frac{\tau}{\kappa}$ constant. Since $\cot\theta$ has range all of $\mathbb{R}$ on domain $(0, \pi)$, find fixed θ so that $\cot\theta = \frac{\tau}{\kappa}$. Define $\vec{u} = \cos\theta T + \sin\theta B$. Then

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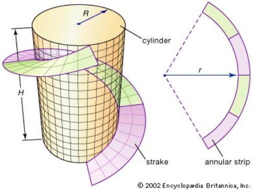
Strake and Auger: Surfaces with Helices



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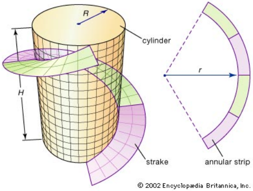

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$\alpha(t) = (3.75\cos t, 3.75\sin t, \frac{31.5}{2\pi}t)$

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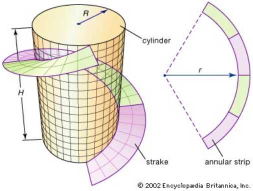

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$\alpha'(t) = (-3.75\sin t, 3.75\cos t, \frac{31.5}{2\pi})$ so

$|\alpha'| = \sqrt{3.75^2 + \frac{31.5^2}{4\pi^2}} \approx 6.26$

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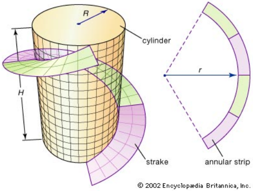

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$$T(t) = \left(\frac{-3.75}{\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}}\sin t, \frac{3.75}{\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}}\cos t, \frac{31.5}{2\pi\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}}\right)$$

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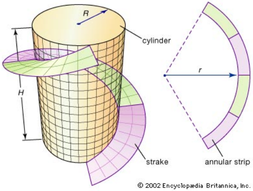

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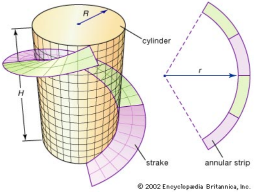
$T(t) = \left(\frac{-3.75}{\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}}\sin t, \frac{3.75}{\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}}\cos t, \frac{31.5}{2\pi\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}}\right)$

$\vec{\kappa}(t) = \left(\frac{-3.75}{\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}^{\,2}}\cos t, \frac{-3.75}{\sqrt{3.75^2+\frac{31.5^2}{4\pi^2}}^{\,2}}\sin t, 0\right)$ $|\vec{\kappa}| = \frac{3.75}{3.75^2+\frac{31.5^2}{4\pi^2}}$

Strake and Auger: Surfaces with Helices



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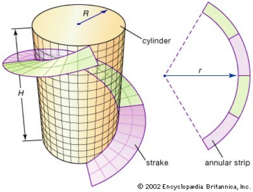
$$r = \frac{1}{\kappa} = \frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75} \approx 10.452$$

inner helix = $(3.75\cos t, 3.75\sin t, \frac{31.5}{2\pi}t)$

inner annulus $\left(\frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75}\cos t, \frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75}\sin t, 0\right)$

Strake and Auger: Surfaces with Helices



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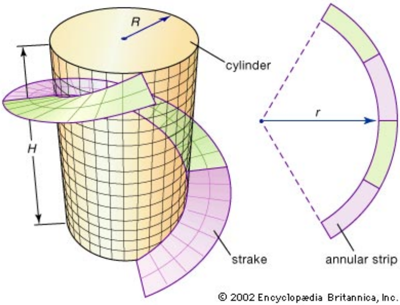
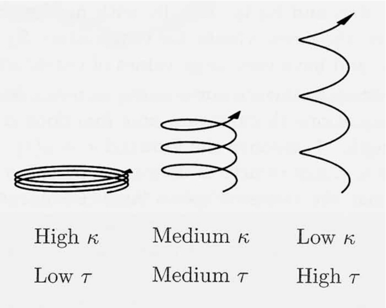
inner helix = $(3.75\cos t, 3.75\sin t, \frac{31.5}{2\pi} t)$

inner annulus $(\frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75}\cos t, \frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75}\sin t, 0)$

outer helix $((3.75 + w)\cos t, (3.75 + w)\sin t, \frac{31.5}{2\pi} t)$

outer annulus $((\frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75} + w)\cos t, (\frac{3.75^2 + \frac{31.5^2}{4\pi^2}}{3.75} + w)\sin t, 0)$





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inner annulus $(\frac{3.75^2+\frac{31.5^2}{4\pi^2}}{3.75}\cos t, \frac{3.75^2+\frac{31.5^2}{4\pi^2}}{3.75}\sin t, 0)$

outer helix $((3.75+w)\cos t, (3.75+w)\sin t, \frac{31.5}{2\pi}t)$

outer annulus $((\frac{3.75^2+\frac{31.5^2}{4\pi^2}}{3.75}+w)\cos t, (\frac{3.75^2+\frac{31.5^2}{4\pi^2}}{3.75}+w)\sin t, 0)$