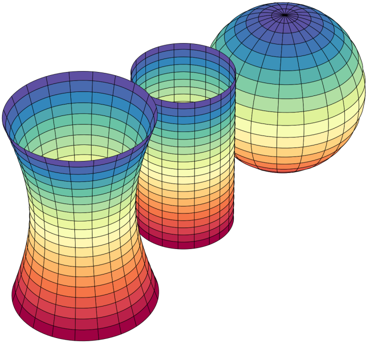
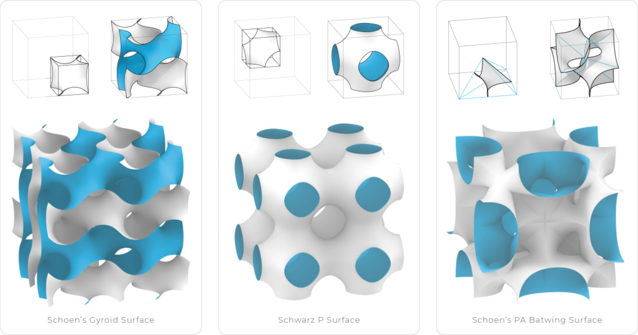
1. According to the reading, one of the greatest achievements of the theory of surfaces was:

a) analytic geometry
b) Gauss-Bonnet theorem
c) classification of surfaces
d) discovering surfaces that stretch the imagination
e) none of the above



2. How long have surfaces been studied?

a) In a very short time period during Gauss' lifetime
b) From at least the Greeks until the 1800s
c) From at least the Greeks until the 1900s
d) From at least the Greeks until now
e) none of the above



https://wewanttolearn.files.wordpress.com/2019/01/tpms-cover-1.jpg?w=1200

3. According to the reading, what does the Gaussian curvature measure?

a) the deviance of a curve on the surface from being a geodesic
b) the deviance of the surface from being a plane at each point
c) the deviance of the surface from being a round earth at each point
d) how curvy Gauss' ear was
e) none of the above

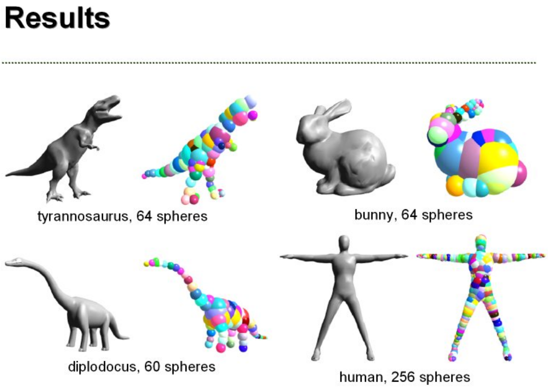
4. For curves we learned that curvature and torsion determine the curve up to rigid motion. What are the corresponding features that determine a surface up to rigid motion?

a) two parameters curvature and torsion
b) one parameter, the Gaussian curvature
c) six coefficients of parametric equations called the first and second fundamental forms, local invariants that are functions of arc length
d) eleven dimensions from string theory
e) need an infinite amount of information to obtain a surface

SURFACE TENSION

5. Is is possible to win an Academy Award (Oscars) for working on surfaces?

a) yes and it has already happened
b) yes but no one has yet
c) no



http://research.microsoft.com/en-us/um/people/johnsny/images/sphapprox_big.jpg

6. According to the reading, surfaces can be represented using

a) 1–8 below
b) all but 1 and 8
c) other

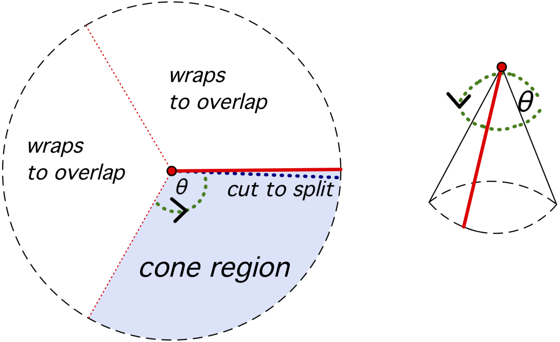
1) if it is surface of revolutions, then by the revolutions that form it
2) analysis
3) algebra
4) geometry
5) polygon meshes
6) physical models and sculptures
7) computer animations
8) soap bubbles

Cones and Cylinders

Maple visualization of rolling a geodesic—cone and cylinder

Cones and Cylinders

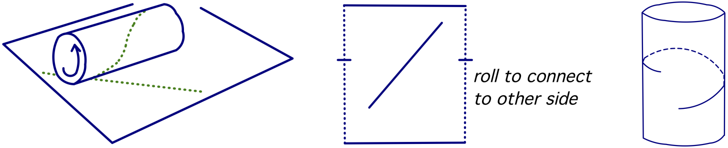
Maple visualization of rolling a geodesic—cone and cylinder
$0 < \text{cone angle} < 2\pi$ variable cone



$\frac{2\pi}{3}$ *cone*

We can vary the angle by changing the cone region before we wrap the rest around (it doesn't have to fit evenly into 2π)

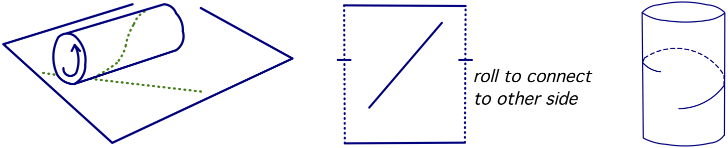
(Intrinsically Straight) Geodesics on a Cylinder



- symmetry and our feet
- rolling arguments (covering arguments in general)

1. Geodesic ever intersect itself?

(Intrinsically Straight) Geodesics on a Cylinder



- symmetry and our feet
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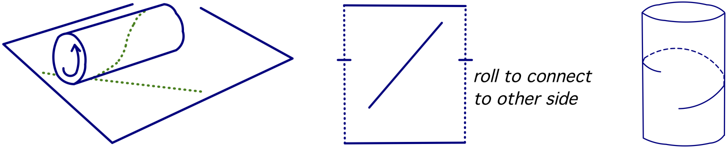
1. Geodesic ever intersect itself? Yes. horizontal circle
2. Shapes?

(Intrinsically Straight) Geodesics on a Cylinder



- symmetry and our feet
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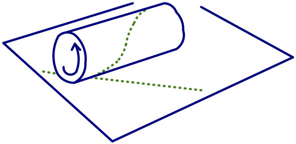
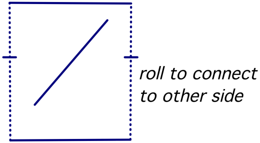
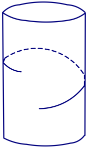
1. Geodesic ever intersect itself? Yes. horizontal circle
2. Shapes? vertical lines, horizontal circles, helices (constant angle is made with the z-axis because it is a straight line on the unrolled cylinder)
3. Straight always shortest distance?

(Intrinsically Straight) Geodesics on a Cylinder







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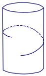
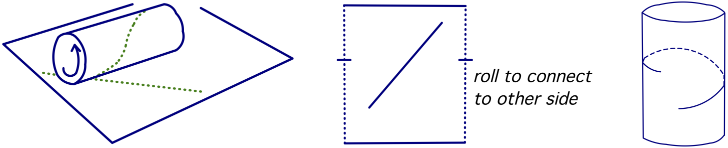
3. Straight always shortest distance? No.  or circle backside
4. Shortest distance always straight?

(Intrinsically Straight) Geodesics on a Cylinder



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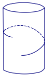
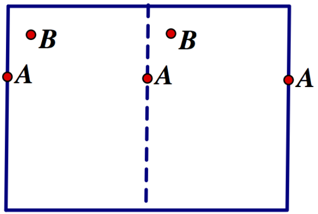
3. Straight always shortest distance? No.  or circle backside
4. Shortest distance always straight? Yes. shortest on cylinder is shortest on covering & hence intrinsically straight on both
5. How many geodesics join 2 points?

(Intrinsically Straight) Geodesics on a Cylinder

5. How many geodesics join 2 points? A 2-sheeted covering:



- Fold a paper in half vertically so you have 2 equal regions
- Label point *A* on each edge at the same height (3 *A*s)
- Choose *B*s not on the same vertical or horizontal line as *A*
- Draw a line between every *A* and every *B*. Marker is best.
- Roll the sheet up so *A*s match & examine the geodesics

(Intrinsically Straight) Geodesics on a Cylinder

5. How many geodesics join 2 points?

horizontal points: 1 (they are part of the same geodesic circle, aside from # times it overlaps or goes around front or back)

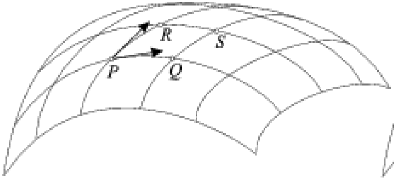
non-horizontal points—keep adding sheets to the covering: ∞ (countably)

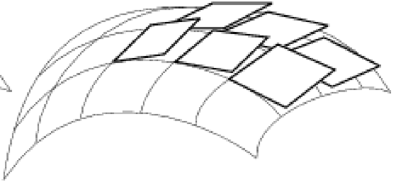
Applications of unwrapping: Surface Area of a Cylinder

- intrinsic
  travel on a straight path until we come back around to where we started and measure that distance
- compute extrinsic surface area using the covering

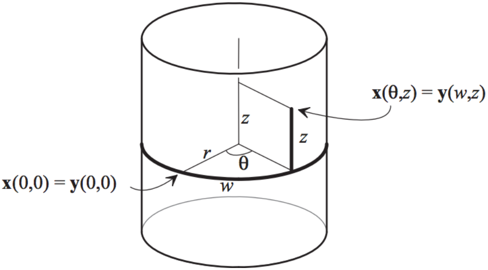
Applications of unwrapping: Surface Area of a Cylinder

- intrinsic
  travel on a straight path until we come back around to where we started and measure that distance
- compute extrinsic surface area using the covering
- later we will compute surface area more generally using the first fundamental form





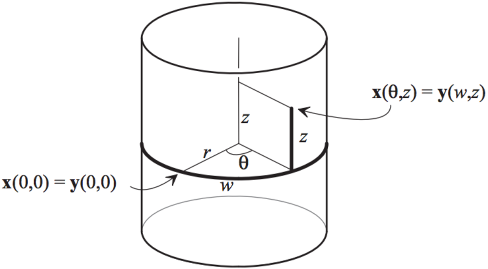
Extrinsic Coordinates on a Cylinder



http://pi.math.cornell.edu/~henderson/courses/M4540-S12/11-DG-front+Ch1.pdf

- Choose (0,0,0), 3 $\perp$ axes, +z as a cylinder height axis
- Let θ be the angle traveled from the origin in the *xy* plane

Extrinsic Coordinates on a Cylinder



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- Choose (0,0,0), 3 $\perp$ axes, +z as a cylinder height axis
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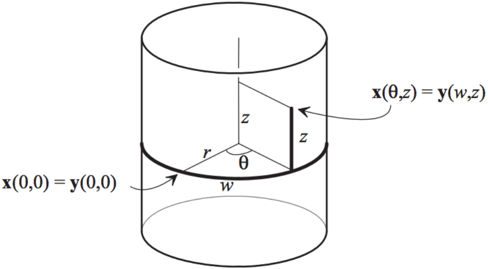
Extrinsic coordinates : $x(\theta, z) = (rcos(\theta), rsin(\theta), z)$.
Equation of cylinder: $x^2 + y^2 = r^2$ in $\mathbb{R}^3$
Compute $T_p$cylinder and U

Extrinsic Coordinates on a Cylinder



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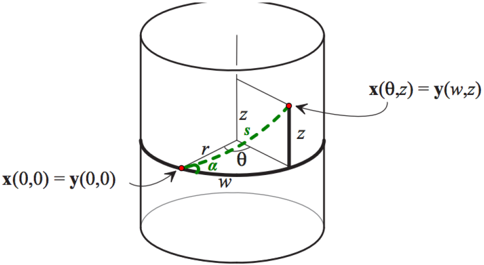
Compute $T_p$cylinder and U
Problem: Bug no awareness of 3-space

Intrinsic Coordinates on a Cylinder



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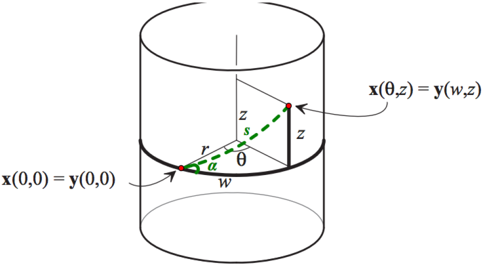

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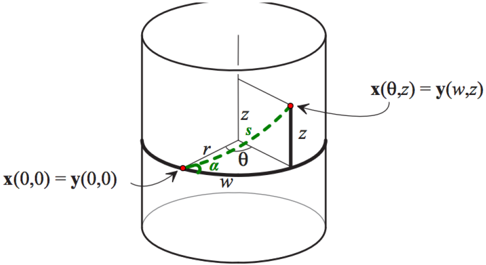
Intrinsic Coordinates on a Cylinder



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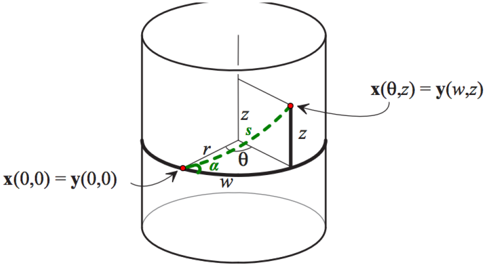
- Choose (0,0) as an intrinsic origin. There is 1 geodesic that will return there, so call that the base curve
- Choose +z as a direction ⊥ to the base curve

Geodesic rectangular coordinates: $y(w, z)$ = walk w units along base curve and turn 90° to positive z and travel z units.

Intrinsic Coordinates on a Cylinder



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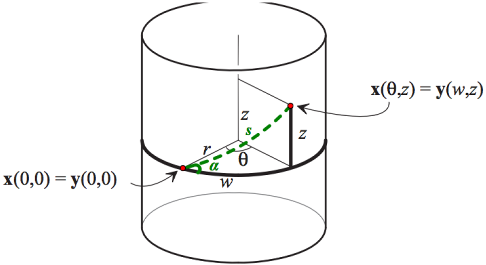
Geodesic polar coordinates: $y(\alpha, s)$ = turn α degrees from the base curve and walk s units along that geodesic

Parameterize $\gamma(s)$ and use s, α, w, z to write equation.

Intrinsic Coordinates on a Cylinder



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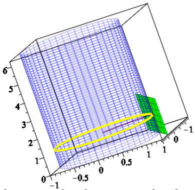
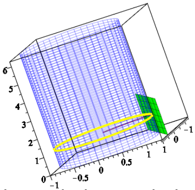
Parameterize $\gamma(s)$ and use s, α, w, z to write equation. Find α?

Curvatures on Surfaces in Extrinsic Coordinates



- Cylinder: **x** (u, v) **=** $(cos(u), sin(u), v)$
- $\vec{x}_u$ and $\vec{x}_v$ are tangent vectors
- The *unit normal* to the surface at a point is $U = \frac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$ determines the tangent plane

Curvatures on Surfaces in Extrinsic Coordinates



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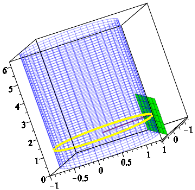
- If $\vec{\kappa}_\alpha$ is the curvature vector for a curve $\alpha(t)$ on the surface then the *normal curvature* is the projection onto U:
$$\vec{\kappa}_n = (U \cdot \vec{\kappa}_\alpha)U$$
- The *geodesic curvature* is what is felt by the bug (in the tangent plane T_pM):

Curvatures on Surfaces in Extrinsic Coordinates



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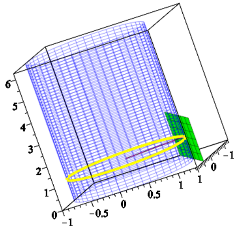
Maple File on Geodesic and Normal Curvatures

adapted from David Henderson



$\vec{\kappa}_\alpha$ pink dashed thickness 1
$\vec{\kappa}_n$ black solid thickness 2
$\vec{\kappa}_g$ tan dashdot style thickness 4

- The *unit normal* to the surface at a point is $U = \frac{\vec{x}_u \times \vec{x}_v}{|\vec{x}_u \times \vec{x}_v|}$
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Commands for Maple File on Curvatures

```
g := (x,y) -> [cos(x), sin(x), y]:
a1:=0: a2:=2*Pi: b1:=0: b2:=2*Pi:
c1 := 1: c2 := 3*Pi:
Point := 2:
f1:= (t) -> t:
f2:= (t) -> 1:

g := (x,y) -> [cos(x), sin(x), y]:
a1:=0: a2:=2*Pi: b1:=0: b2:=Pi:
c1 := 1: c2 := 3:
Point := 2:
f1:= (t) -> t:
f2:= (t) -> sin(t):
```