Exam 1: Curves

It is time for our first exam in order to be sure that everyone reviews some of the fundamental concepts before we move on to surfaces.

At the Exam

- You may make yourself some reference notes on the very small card I hand out. The mini reference card must be handwritten. Think of the card as a way to include some important concepts, computations, or derivations that you aren't as comfortable with. You won't have room for much, so you should try to internalize as much as you can.
- One scientific calculator or graphing calculator allowed (but no cell phone nor other calculators bundled in combination with additional technologies). I don't see that you would need this, but I know some people like to have it with them.
- You may have out food, hydration, ear plugs, or similar if they will help you (however any ear plugs must be standalone–no cell phone, internet or other technological connections)
- There will be various types of questions on the exam and your grade will be based on the quality of your responses in a timed environment.
  - Part 1: Fill in the blank/short answer
  - Part 2: Calculations and Interpretations
  - Part 3: Short Derivations/Proofs
- Partial credit will be given, so (if you have time) showing your reasoning or thoughts on questions you are unsure of can help your grade.

Review

I suggest that you review your class notes, the calendar and class activities page (that has the slides and clicker questions), and ASULearn solutions.

Part 1: Fill in the blank/short answer There will be some short answer questions, such as:

- definitions related to any of the items in the glossary on curves
- parametrizations, curvature or torsion of "basic" curves such as a circle, line, plane curves y=f(x), or a helix or strake
- questions similar to previous clicker questions, the matching or other activities from class where you fill in a blank instead. For instance,
  - $-\kappa T + \tau B =$ ____________________
  - $N' =$ ____________________
  - unit vector that lies along the direction which the curve is currently bending in = ____________

  As you can see here, there is often more than one answer possible for fill in the blank questions: choose one response. Full credit responses demonstrate deep understanding of differential geometry. For instance, here you could fill in $-\kappa T + \tau B = N'$ for the first response, $N' = -\kappa T + \tau B$ for the second, and $\frac{\frac{T'(t)}{|\alpha'(t)|}}{\left|\frac{T'(t)}{|\alpha'(t)|}\right|}$ for the third, among other possible responses. On the other hand, responding with $N' = N'$, while a true statement, doesn't demonstrate deep understanding of differential geometry. Informal responses are fine as long as they are correct and demonstrate understanding of the material from class and homework.

Part 2: Calculations and Interpretations There will be some by-hand computations and interpretations, like

- Solving for the scalar curvature of a plane curve
- Finding $T(t), T(s), \vec{\kappa}$, and κ curvature for a curve
- Finding B and τ, given T and N
- Finding N, given T
- Interpreting results and applications of the Frenet Frame, like recognizing that a line is the shortest distance curve between two points in Euclidean geometry, a $\tau = 0$ curve is planar, a $\kappa = 0$ curve is a line, a curve with constant positive scalar curvature that is planar is part of a circle, a curve with constant $\frac{\tau}{\kappa}$ is a cylindrical helix...

Part 3: Short Derivations/Proofs Review and be able to prove the following:

- Prove that for a regular curve, $s(t)$ has an inverse (and showing how the mean value theorem applies).
- Prove the derivative of a unit vector $\vec{u}$ is perpendicular to the original vector if neither are $\vec{0}$.
- Prove B is a unit vector. You may assume that T and N are unit length and perpendicular to each other, and that B is defined in terms of them.
- The proofs of the Frenet equations. You would be given one short part and some underlying assumptions:
  - Prove B' has no component in the T direction. You may assume that the Frenet equation about T' holds but not the other two. You may also assume cross product relationships and dot product relationships between T, N and B.
  - Prove that B' has a $-\tau$ component in the N direction. You may assume that the Frenet equation about T' holds but not the other two. You may also assume cross product relationships and dot product relationships between T, N and B.
  - Prove that N' has a $-\kappa$ component in the T direction. You may assume that the Frenet equations about T' and B' hold but not the one about N'. You may also assume cross product relationships and dot product relationships between T, N and B.
  - Prove that N' has a τ component in the B direction. You may assume that the Frenet equations about T' and B' hold but not the one about N'. You may also assume cross product relationships and dot product relationships between T, N and B.
- Prove that $\alpha(s)$ is a line $\Leftrightarrow \kappa = 0$.
- Prove the Darboux derivations from homework 3 and Feb 6 clicker question.

We covered the proofs above in classes. You should know the results of other statements we proved in class, which could be asked about in the first two sections of the exam, but I won't ask you for any other complete proofs, other than the ones above.

I want you to understand and I am happy to help!