Futurama Math: Functional Literacy in "Quids Game" Activity Sheet

by Steven Zides, Wofford College, and Sarah J. Greenwald, Appalachian State University

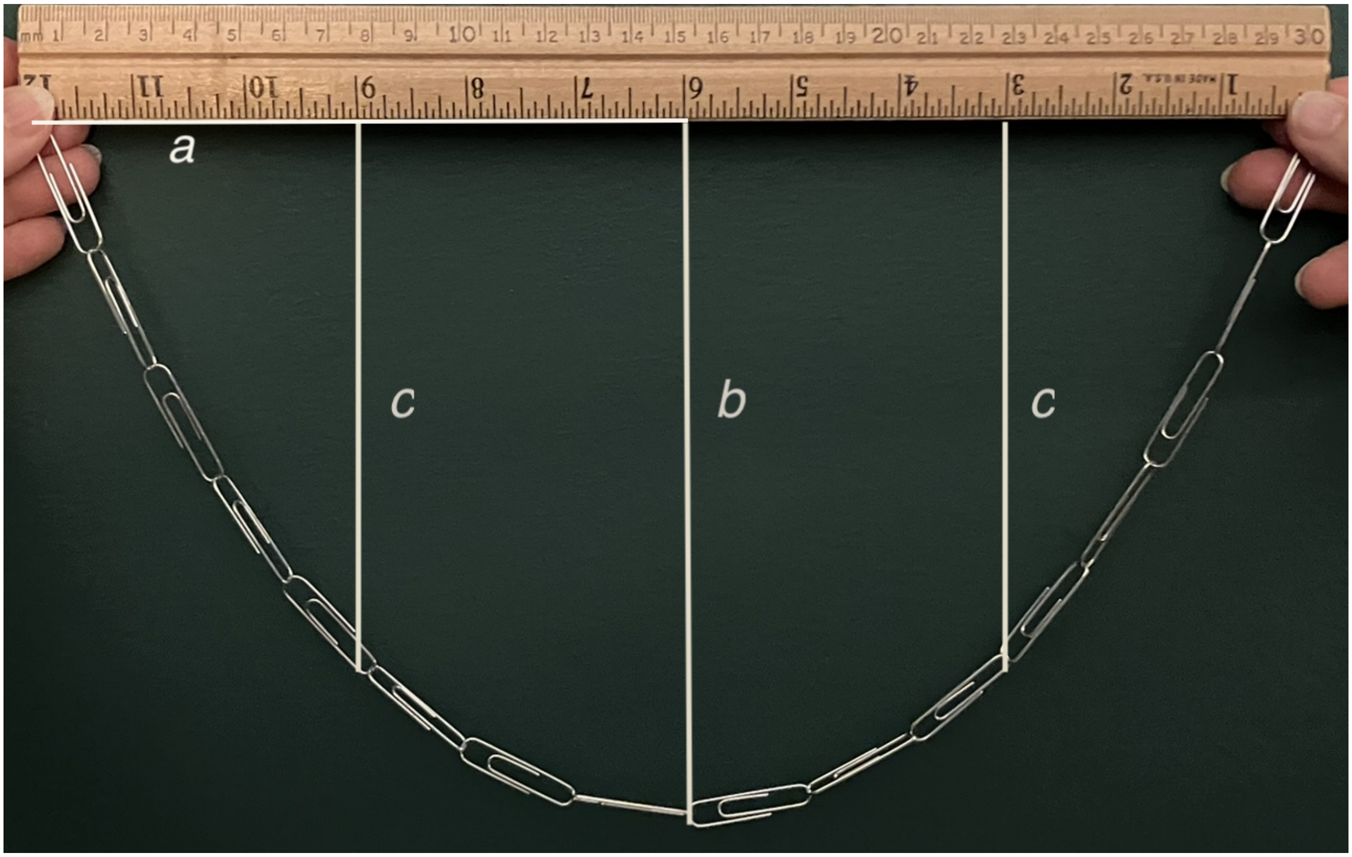
equipment for each group: two 12-inch or 30-cm rulers, about 10–15 paperclips, and access to Desmos.
optional equipment: 1 yardstick or meterstick

Futurama follows Fry, who fell into a cryogenic chamber and woke up 1000 years later. In the twelfth season's episode "Quids Game" (Season 12, Episode 2), Fry and everyone attending his birthday party are kidnapped by aliens and forced to participate in life-or-death party games. In one game, Fry and his friends are forced to hang arm in arm (like the famous game, Barrel of Monkeys) in one of two long chains. To make it more challenging, the aliens place a pool of dissolving liquid below.

One of the math-savvy characters, Bubblegum Tate makes the comment, "Now, hold on a minute. According to the laws of calculus, we might just be able to link chains to form a stable structure…" which might make it possible to hang longer without falling.

1. Let's try to investigate the curve that Bubblegum Tate mentions by first modeling the situation mentioned in the show. Start with a ruler held horizontally, upside down, so that the numbers are on the bottom. Hook together two sets of 5 large paper clips (or two sets of 8 small paper clips). Have a group member hold each set of paper clips on either edge of the ruler, near the bottom corner closest to the floor. This is essentially the scene portrayed in the episode. Unfortunately, in the episode Bubblegum Tate's arm gives out before he can enact his plan.

2. However, you and your group can finish his work by now connecting the bottom of both sets of paperclips. This should give you a curve that hangs from the two bottom corners of the ruler. You might want to take a picture of the curve with your phone for later use.



3. Now we need to collect some data. To keep things simple, we envision an imaginary y-axis to pass through the center of the ruler, with $y = 0$ starting at the bottom edge closest to the floor. We call the horizontal distance between the center of the ruler and its left and right edges a, which is probably either 6 inches or 15 cm. Next measure the "drop" of the curve, from the bottom edge of the ruler to the center of the paperclip chain, at $\frac{1}{4}$ the way out along its length and then again at $\frac{3}{4}$ the way out along its length. If you are holding the ruler correctly, you should get the same value, since the paperclip chain should be symmetric. If not, try to hold the ruler horizontally and measure these "drops" again. Call the value of the "drop" you get at the $\frac{1}{4}$ and $\frac{3}{4}$ locations c.

4. Finally, measure the "drop" of the curve, from the bottom edge of the ruler to the center of the paperclip chain, at the center of the ruler and call this value, b. Since this should be the lowest part of your curve, the value for b should be bigger than your value for c. Write down your collected values: $a =?$, $b =?$, $c =?$

5. Looking at the picture of your curve and the data you collected, do you think this curve could be a semicircle? Why or why not?

6. Looking at the picture of your curve and the data you collected, do you think this curve could be some kind of cosine function? Why or why not?

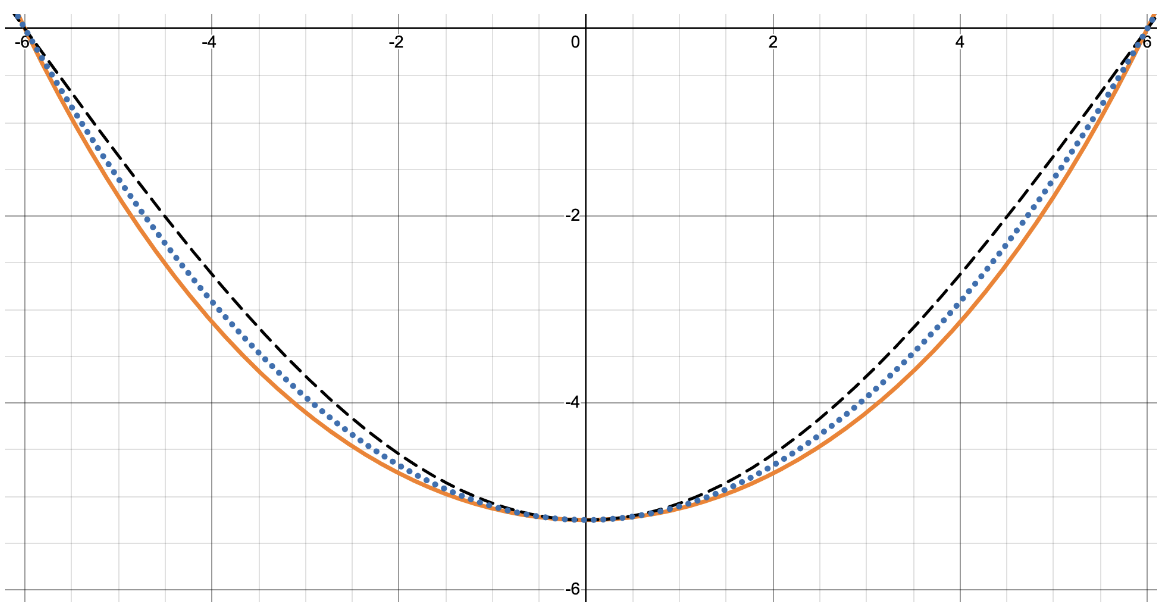
7. To get a better sense for the cosine curve, let's use Desmos (the online web graphing program) to model the cosine function $f(x) = (-b)\cos((\frac{\pi}{2a})x)$, where a and b are the numerical values from earlier. What does Desmos predict for the "drop" in your cosine curve at $\frac{1}{4}$ the way out location, which is the value $f(\frac{a}{2})$? Does that match up with your value c from earlier?

8. Looking at the picture of your curve and the data you collected, do you think this curve could be some kind of parabola? Why or why not?

9. To get a better sense for the parabola, let's use Desmos to model the parabola function $g(x) = (\frac{b}{a^2})(x+a)(x-a)$, where a and b are the numerical values from earlier. What does Desmos predict for the "drop" in your parabola at $\frac{1}{4}$ the way out location, which is the value $g(\frac{a}{2})$? Does that match up with your value c from earlier?

10. Unfortunately, most people find that neither the cosine curve nor parabola match the natural curve of the hanging chain, which seems to drop off faster than either of these other two functions. Luckily, mathematicians are very crafty and in the late 1700 century they found an equation that correctly matches the curve, $y = (-b) + k(\frac{1}{2}(e^{\frac{x}{k}} + e^{-\frac{x}{k}})) - k$, where b is our data value from earlier and k is something that needs to be estimated numerically, to fit the endpoints of the curve. To get a better sense for this new curve, let's use Desmos, one more time, to model this new function, $h(x) = (-b) + k(\frac{1}{2}(e^{\frac{x}{k}} + e^{-\frac{x}{k}})) - k$, where b is the numerical value from earlier. To get the function to match your endpoints, use the Desmos slider ability to get a k value which matches the zeros (or endpoints) of the other curves you have looked at so far. What does Desmos predict for the "drop" in this new curve at $\frac{1}{4}$ the way out location, which is the value $h(\frac{a}{2})$? Does that match up with your value c from earlier?

11. As a point of comparison, here are the three curves graphed all at once in `https://www.desmos.com/calculator/1nlt3zhhyj`, for the following set of hypothetical data, $a = 6, b = 5.25, c = 4.125$. The steepest dashed curve represents the cosine function, the middle dotted curve is the parabola, and the solid bottom curve is our new curve, which was named the catenary.



12. In the *Futurama* episode, if you watch carefully, you can see Bubblegum Tate write down something very similar to the catenary equation we worked with above, the $h(x)$ function with two exponential terms. Interestingly enough, in the episode he also equates this to something of the form $k\cosh(\frac{x}{k})$, which must be some kind of shorthand notation. In other words, mathematicians must have decided to define $\cosh(x)$ to represent $\frac{1}{2}(e^x + e^{-x})$. But why pick $\cosh(x)$, which looks very similar $\cos(x)$, when our new curve, the catenary, does not have a wave-like periodic shape? To see why, first take the first through fourth derivatives of the expression $\frac{1}{2}(e^x+e^{-x})$ and evaluate these expressions at $x = 0$.

13. Then consider specifically the second and fourth derivatives. Do any other common functions have similar characteristics?

14. Now consider the first derivative—what might be a good name for this first derivative expression? Then revisit question 12.

15. As a final optional activity, collect all the paperclips together and make a very long catenary curve, maybe 3ft (or 1m) at its lowest point, hanging from the original ruler, keeping the a value the same. Then use a yardstick or meter stick to try to take a similar set of data. This could be challenging (but fun) since the b and c values will now be quite large.

16. Do you think the cosine curve and parabola curves will do a better job matching the catenary curve for this extended length? To check this, enter this new set of data into Desmos and see what all three curves give you. In other words, for this very long curve how close are $f(\frac{a}{2}), g(\frac{a}{2})$ and $h(\frac{a}{2})$?