Activity Sheet: Hypatia and Archimedes' *Dimension of the Circle*

Hypatia is the first woman mathematician about whom we have either biographical knowledge or knowledge of her mathematics. Hypatia developed commentaries on older works, probably including those by Ptolemy, Diophantus, and Apollonius, in order to make them easier to understand. Hypatia was probably the first woman to have a profound impact on the survival of early thought in mathematics

Since Hypatia lived so long ago, it is hard to know exactly what she worked on, although we do have some specific historical evidence of her mathematics and an account of her horrible death. We know that original scholarship was not Hypatia's focus. Together with her father Theon, she helped preserve some of the treasures of ancient Greek mathematics and astronomy. While she cannot compare in originality with the mathematicians that she wrote commentaries on, her reputation as a teacher and scholar is secure.

Hypatia may have written a commentary on Archimedes' *Dimension of the Circle*. Wilbur Knorr, a math historian, identified a certain style of writing that he attributes to Hypatia. He learned new languages so that he could analyze different versions of Archimedes' *Dimension of the Circle* in Hebrew, Arabic, Latin and Greek. Although there is no direct evidence of the existence of commentaries developed by Hypatia on Archimedes' work, Knorr suggests that Hypatia's influence can be found there. As research and analysis of ancient texts continues, we may indeed learn more about Hypatia's mathematical contributions. We will explore mathematical ideas from Archimedes' *Dimension of the Circle* and in this way we will see some mathematics that Hypatia might have worked on.

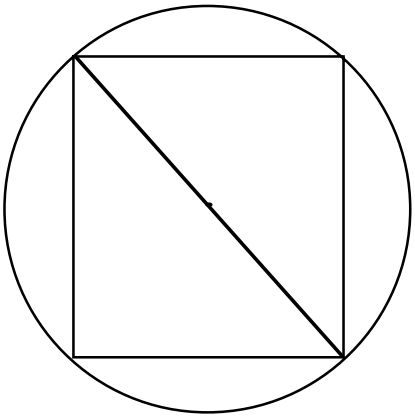
Archimedes worked to establish a very good estimate of the value of the ratio of circumference to diameter that we today call "π", and he proved the following theorem: The area of any circle is equal to the area of a right-angled triangle in which one of the sides about the right angle is equal to the radius and the other to the circumference of the circle.

1. Archimedes' theorem states that for any circle, one-half the perimeter times the radius is equal to the area. Using formulas for the area and perimeter (circumference) of a circle, in terms of the radius, show that this statement is true.

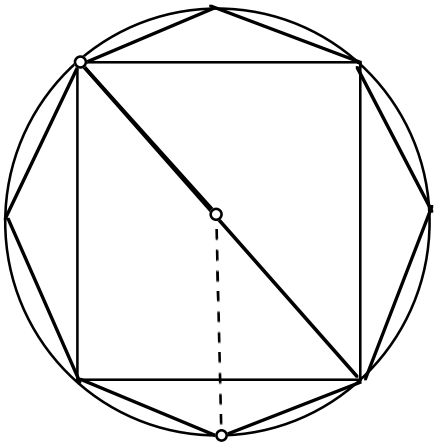
Since Archimedes and the mathematicians who later wrote commentaries on his work, such as possibly Hypatia, were not working with the formulas that we use today, they

were interested in proving this statement. Archimedes proved the theorem by inscribing and circumscribing polygons about a circle. Here are some of the constructions related to his clever proof.

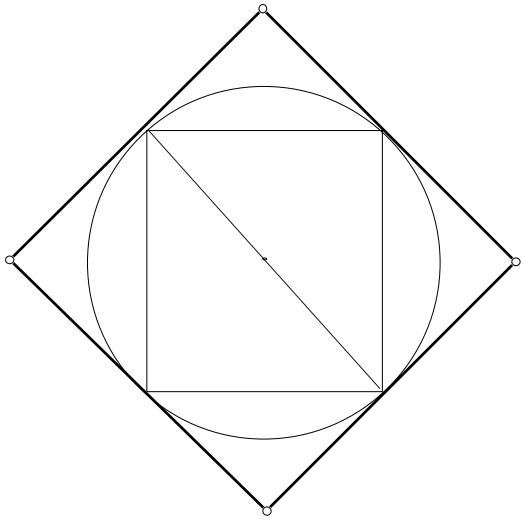
2. Construct and find the area of the square inscribed in a circle of diameter 6 inches as shown below. Archimedes knew the Pythagorean Theorem and several useful facts about circles and squares that you already know. State any facts that you use.



3. If we bisect the arcs formed by the inscribed square, then we will obtain four new points. We can connect these points and the corners of the square with straight lines in order to obtain the octagon in the picture below. Find the area of the inscribed regular octagon. Give your answer both in exact radical notation and approximated to 4 decimal places.



4. Find the area of the square circumscribed about the same circle we started with in question 2.



5. What bounds have you now found for π? You should approximate your results to four decimal places.

6. Calculate the area of the your circle with diameter 6 inches, using 3.1416 as the approximation of π. Compare this value to the approximations in questions 2-4.