TAKING FULL ADVANTAGE OF SKETCHPAD 4 – WHY YOU CAN'T LIVE WITHOUT IT

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Introduction

Sketchpad [6] is a dynamic visualization software package that encourages mathematical discovery. Students manipulate geometric sketches that they have constructed themselves by movements that do not change the dependency relationships. This enables them to discover invariants and make conjectures. In this way they develop an ownership of the material and experience the excitement of discovering mathematics.

The latest version of Sketchpad features enhancements such as built-in web integration, improved sample explorations, and manipulation of functions that make it an effective learning environment in a greater variety of classes [5]. We will look at examples of Sketchpad 4 explorations that were not possible in older versions **[4]**, along with the advantages, limitations and effects on teaching and learning in the classroom.

Calculus Add-On Module

The *Exploring Calculus with The Geometer's Sketchpad* add-on curriculum module [3] is a wonderful tool that helps students visualize calculus concepts. In the paragraphs below we outline some of the uses and innovations of the Exploring Calculus Module.

We found the Limits folder to be quite useful in illustrating the ideas behind limits and the epsilon-delta definition of limit. One file allows students to investigate right and left hand limits for several functions involving point discontinuities, such as sin(x)/x or equations involving absolute values.

The EpsilonDelta.gsp file, within the Limits folder, illustrates the epsilon-delta definition of the limit. This concept seems to be a stumbling block for many of our students and having a geometric explanation certainly seems to help. Students can set an "epsilon window" along the y-axis and then experiment with "delta windows" along the x-axis, via sliders (see Figure 1), in order to determine limit relationships. This allows us to emphasize what the definition "for every epsilon there exists a delta …" means. Several students have commented that this helped them have a deeper understanding of the definition of a limit.
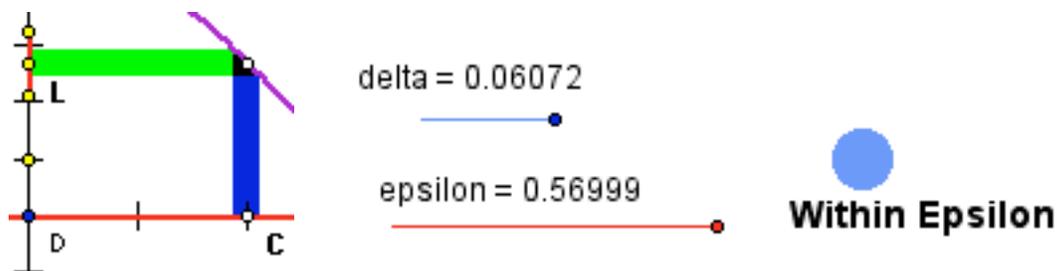
Figure 1. Epsilon-Delta Sliders

The Applications_of_Derivatives folder is enlightening as well. The RelatedRates.gsp file allows students to animate related rate problems. For example, students can visualize what happens to the area of a square as its side length increases at a constant rate or see what happens to the side length as the area of the square increases at a constant rate. The MaxMin.gsp file explores traditional maximization and minimization problems. But now students can graph the quantity that they wish to optimize, via sliders again, and make conjectures for the answer. These files allow students to conjecture what relationships exist and then the students can use calculus to prove or disprove their conjectures. The files within this folder are a wonderful supplement that gives the student something "concrete" to visualize.

We also found the RelatedRates.gsp file quite useful for illustrating the ideas, as well as a good source of extra examples. We would use the problems within Sketchpad as our in-class examples and thus Sketchpad could be used without affecting overall class time. Without this file, we cannot give students a true picture of related rate problems. Instead, visualizations are left to their imaginations and, if their understanding of the concept is minimal, their imaginations may not provide good examples. This can lead to fundamental misunderstandings of this topic. The use of Sketchpad seemed to help dissipate some of these misunderstandings.

The Antiderivatives_Areas_Integrals folder also allows new "spins" on traditional ideas. We used the RectangleDemo.gsp and RiemannSums.gsp files, which allow students to receive an excellent visualization of the dynamic nature of the Riemann integral and give the teacher new tools to discuss the differences between right-hand, left-hand, midpoint, upper, and lower sums. The drawings are crisp, as opposed to most hand written drawings, and the students could clearly see the geometric ideas behind the definite integral. Other files allow the user to investigate the relationships between the definite integral and the signed area "under" a curve.

Other folders seem to offer a wide range of visualizations; however, we have not, at present, incorporated any of these into our classes. Some files included ideas beyond the scope of our courses. Because our classes are already rushed, we felt that other files, while interesting, did not offer enough extra insight to justify lost class time. As with any additional tool, the teacher must decide the rewards versus the time investment for each file within the module. For our course the tools used fit naturally within the course and were incorporated in such a way that lost class time was minimal.

Poincare Disk Exploration of Hyperbolic Geometry

Non-Euclidean geometry is a popular topic in classes ranging from the middle grades level to the college level. While spherical geometry is easy to explore using hands-on models, hyperbolic geometry often seems artificial and confusing for students because models that do exist are not easy to manipulate during exploratory activities. Limited capabilities for a Poincare disk exploration did exist in Sketchpad 3, but in order to investigate this model of hyperbolic geometry in Sketchpad 3, one had to find and download or create a special script and then put the script on every computer. In Sketchpad 4, students can immediately begin investigating hyperbolic geometry by opening a built-in sample sketch that is located within the Sketchpad folder (Sketchpad/Samples/Sketches/Investigations/Poincare_Disk.gsp).

Students can easily create hyperbolic lines, triangles, and circles and measure hyperbolic lengths and angles in order to evaluate the validity of Euclid's postulates and propositions in this model. For example, students create a hyperbolic triangle, measure the angles, and then calculate the sum of the angles in the triangle (see Figure 2). We then ask students to investigate how small and large the sum of the angles can get. Since the dependency relationships are preserved when students move a vertex of the triangle, students quickly answer this question by investigating various triangles. This allows them to develop intuition as well as a connection to the hyperbolic disk that was not previously possible. In addition, they can easily work side-by-side with similar Euclidean constructions (see Figure 3).

$$m\angle BAC = 24.7°$$
$$m\angle ACB = 32.1°$$
$$m\angle CBA = 22.8°$$
$$m_1+m_2+m_3 = 79.51°$$
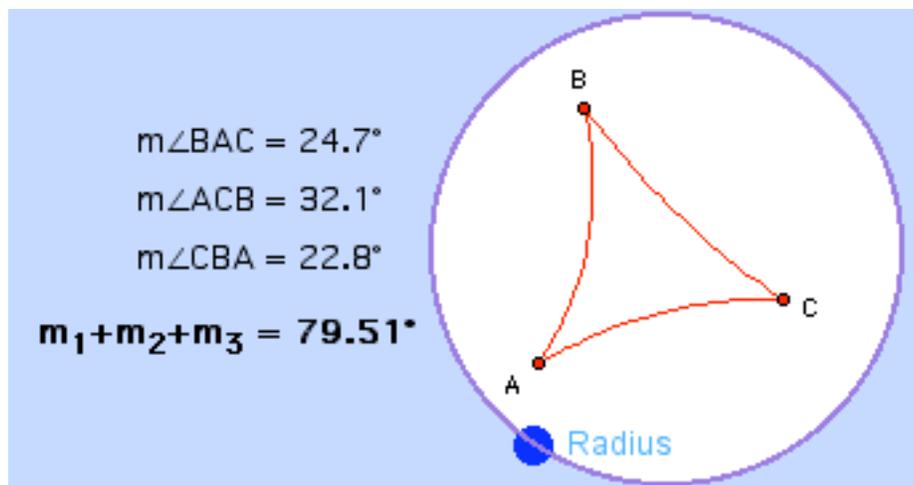
Figure 2. **Sum of the Angles of a Poincare Disk Triangle**

Students may encounter certain difficulties with these types of explorations. In the Poincare disk model, the boundary of the disk represents infinity, and yet, students are able to drag the vertices of a hyperbolic triangle beyond the boundary. In addition, the discrete nature of Sketchpad can be problematic when students wish to move a vertex in order to achieve a certain measurement. Often they will be unable to achieve desired measurements since the points do not move continuously. In addition, rounding errors

can create confusion. For example, in Figure 2, Sketchpad calculates 79.51 as the sum of the angles in the hyperbolic triangle while the sum resulting from the three listed angles is 79.6. Rounding problems can also occur in Euclidean geometry. For example, in order to investigate the Pythagorean Theorem, students create a right triangle and then calculate $a^2 + b^2 - c^2$. Yet, sometimes, as in Figure 4, the calculation of $a^2 + b^2 - c^2$ yields a non-zero answer instead of the expected answer of zero (in Euclidean geometry).

Figure 3. Sum of the Angles of Euclidean and Hyperbolic Triangles

These difficulties can be turned into beneficial educational lessons. The boundary problem in the Poincare disk model can be used to illustrate the difficulty in creating completely accurate models. Students who learn about the discrete nature of the program develop a greater appreciation for and understanding of software issues. In addition, activities that result in rounding errors in Sketchpad can be used to highlight the dangers of blindly accepting results on a calculator or on a computer.

Figure 4. Pythagorean Theorem Rounding Problems in Euclidean Geometry

The benefits of these explorations far out weigh any difficulties and are well worth the class time spent. In-service middle grades teachers, liberal arts students, and undergraduate mathematics majors find that this built-in exploration is invaluable for their understanding of hyperbolic geometry and they enjoy the investigations.

Conclusion

These are just a few of the many possible beneficial Sketchpad 4 activities. Students find Sketchpad easy to learn and use and they enjoy the discovery process and dynamic explorations. Tactile and visual learners especially benefit from Sketchpad activities.

The new capability to define functions and then evaluate, graph, and differentiate them makes Sketchpad 4 a perfect learning environment for algebra [2] and calculus [3], as well as geometry [1, 4, 7]. There are numerous built-in explorations, classroom worksheets, and add-on modules [5] available for teachers to take into their classrooms. Hence, teachers can use these with minimal time investment for preparation. In addition, JavaSketchpad is now built right in to Sketchpad 4 so that one can save files from the pull down menu in order to post them on the Web. In Sketchpad 3 one had to use a special converter. Sketchpad users can post interactive Web activities that can be accessed by anyone with a Java-enabled browser. Hence, teachers can reinforce ideas covered in class by posting JavaSketchpad activities for those who do not have Sketchpad at home.

Sketchpad provides a rich mathematical environment that promotes conjecture and discovery and enhances student comprehension. This is exactly the reason that we use Sketchpad in the classroom, and this is the response we hope for in our own students.

References

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