In Depth Mathematics

One article, entitled “Graduate School Success of Black Students from White Colleges and Black Colleges,” published in 1977, did not employ the most appropriate statistical technique. The question Hrabowski was trying to answer was whether or not African-American students from historically Black colleges and African-American students from predominantly white colleges go on to graduate from Doctoral programs. In this article, he used an F-test to analyze the proportion of graduates from predominantly Black colleges and predominantly white colleges who went on to graduate from Doctoral programs. The F-test is a technique that is usually used for testing numerical data, not proportional data [3]. Proportional data represents percentages or proportions of a sample from a particular category; in this case the categories are graduated and not graduated. A better test for analyzing the categorical data Hrabowski had would have been a X^2-test (chi-squared test). A X^2-test has three general uses, which are explained below:

1. Goodness of fit
2. Testing for independence
3. Test of homogeneity

The X^2-test for goodness of fit is used when determining if the proportions of the samples agree with a specified distribution. The X^2-test of independence is used when subjects in a single population are classified according to two different variables. It is used to determine if there is any dependency between the two categorical variables. The X^2-test of homogeneity is computed in a similar manner to the X^2-test of independence, except that the X^2-test of homogeneity is used when there are several populations and one variable [3]. For the data he had, Hrabowski should have used the X^2-test of homogeneity. The X^2-test for homogeneity is used to determine whether the two populations, students from Black colleges and students from white colleges, have similar or homogeneous distributions with respect to some categorical variable, in this case graduating from a Doctoral program.

In the article, the data is arranged in a contingency table. The rows of the table generally contain the groups being compared and the columns typically contain the different categories. The test statistic is $X^2 = \Sigma \; [(o_i - e_i)^2 / e_i]$, where o_i is the observed value and e_i is the expected value. The expected value is found by computing $e_i = RC/n$, where R is the row total, C is the column total, and n is the total number of observations (in this case students). The expected value gives the number one would expect to see if there were no differences in the two groups. For this case the expected value gives the number of students one would expect to see if graduation rates were not affected by going to a Black school or a white school.

The table below contains the actual numbers from Hrabowski's article, and the values in the table are the observed values.

Graduation Rates in Doctoral Programs among Black Graduate Students by Type of College

Group	Graduated	Not Graduated	Total
A (Students from black colleges)	24	39	63
B (Students from white colleges)	10	30	40
Total	34	69	103

In the following table are the expected values for the table of observed values above, calculated using the formula presented above.

Expected Graduation Rates in Doctoral Programs among black Graduate Students by Type of College

Group	Graduated	Not Graduated	Total
A (Students from black colleges)	20.80	42.20	63
B (Students from white colleges)	13.20	26.80	40
Total	34	69	103

For the observed values and the expected values in the two tables:

$X^2 = (24 – 20.80)^2 / 20.80 + (39 – 42.20)^2 / 42.20 + (10 – 13.20)^2 / 13.20 + (30 – 26.80)^2 / 26.80$. Therefore $X^2 = 1.89$ [1].

The degrees of freedom, denoted df, is equal to (r –1)(c – 1), which in words is the number of rows minus one, times the number of columns minus one. Assuming the total numbers are fixed, the df represents the number of cells that could vary in the table before dependency occurs. For the two by two table, the df = (2 –1)(2 – 1) = 1, which means only one cell can be manipulated because the other cells are dependent on the value in that one cell. From the X^2 table found in most statistic textbooks, but is on page 897 of <u>Exploring Statistics: A Modern Introduction to Data Analysis and Inference</u>, 2nd Edition by Larry J. Kitchens, it is determined that $.200 < p < .100$. To find the p value one would go to a X^2 table, find the number of df, look in that row for the value of X^2, and then determine the value of p. If the value of X^2 falls between two p values, p is placed between the lesser and greater values as shown previously. The variable p stands for the level of significance. A smaller p indicates more significance, and the accepted small p is $p < .05$. The p value of $.200 < p < .001$ means that there is not a significant difference between the proportion of students from black colleges graduating from Doctoral programs to the proportion of black students from white colleges graduating from Doctoral programs [2]. Hrabowski came to the same conclusion with the F-test, but using the F-test could have led him to a different conclusion.

The second article containing statistics that was analyzed and critiqued was published in 1995 and titled “Enhancing the Success of African-American Student in the Sciences: Freshmen Year Outcomes,” he performed tests that were judged to be more appropriate for the question he was asking as well as the data available. In the article Hrabowski explained that a comparison was made between the students in the Meyerhoff Program at UMBC and a sample of students who were going into mathematical and science fields.

To perform an accurate analysis of the effects of the treatment, in this instance the Meyerhoff Program, comparisons were done on characteristics shared by both groups that could affect the end results.   Hrabowski performed comparisons on SAT math and verbal scores, high school grade point averages, gender, and the number of science course credits earned during the freshman year. The science courses included chemistry, physics, biology, math, engineering, and computer science courses. The test Hrabowski used was a two-sample t-test. To check the values he calculated, the two-sample t-test has been reconstructed.

The statistics necessary for conducting a two-sample t-test include: $\boldsymbol{x}$ = the mean of the sample, _ = the population mean, n = the number in the sample, and s = the sample standard deviation (sd). The test statistic for the t-test is: $t = (\boldsymbol{x}_1 – \boldsymbol{x}_2) / \sqrt{}\ \{(s_1^2 / n_1) + (s_2^2 / n_2)\}$. From the table in his article that compared the SAT Math scores for the Meyerhoff students, denoted by the subscript 1, and the sample students, denoted by the subscript 2, $\boldsymbol{x}_1$ =635.7, s_1 = 59, n_1 = 69, $\boldsymbol{x}_2$ = 607.7, s_2 = 46.3, and n_2 = 43. Therefore $t = (635.7 – 607.7) / \sqrt{}\ \{(59^2 / 69) + (46.3^2 / 43)\}$, which equals 2.80 with $p < .01$. This number was not the number Hrabowski reported in his article. It was hypothesized that the reason there was a difference in the two numbers was because Hrabowski used a pooled t-test, which is a special case of the two-sample t-test. A pooled t-test is very similar to a regular t-test, but it is calculated based on the assumption that the population standard deviations, $\_{1}$ and $\_{2}$, are equal. This means that the sample standard deviations, s_1 and s_2, can be combined into one estimation of the joint population standard deviation, _. The equation for a pooled t-test is $t = (\boldsymbol{x}_1 - \boldsymbol{x}_2) / s_p \sqrt{}\ \{(1 / n_1) + (1 / n_2)\}$, where s_p = the pooled standard deviation. To find the pooled standard deviation, $s_p = \sqrt{}\ \{[(n_1 - 1)\ s_1^2 + (n_2 - 1)\ s_2^2] / (n_1 + n_2 - 2)\}$. For the numbers in Hrabowski's article $s_p = \sqrt{}\ \{[59^2 (69 - 1) + 46.3^2 (43 - 1)] / (69 - 43 + 2)\}$, which means $s_p = 54.50$. When the pooled t-test was calculated, $t = (635.7 - 607.7) / 54.50 \sqrt{}\ \{(1 / 59) + (1 / 43)\}$, t was found to be equal to 2.64 with $p < .01$, which was the same value Hrabowski found. In fact, Hrabowski did use the pooled t-test [4].

In the past, it was common to use the pooled t-test because the df is easier to calculate. Modern practice, especially within the past few years, is moving away from the pooled t-test. This is due to the widespread use of computers and the difficulty of verifying or testing the assumption that the population variances and the population standard deviations are the equal. Many people continue to use the pooled t-test because that is what was taught to them [2]. Why Hrabowski used the pooled t-test is not known, but one may guess that it was because the df was easier to calculate than that of a regular t-test. The df of a pooled t-test = $n_1 + n_2 - 2$ and the df of a regular t-test = $\{(s_1^2 / n_1) + s_2^2 / n_2)\}^2 / \{[(s_1^2 / n_1)^2 / (n_1 - 1)] + [(s_2^2 / n_2)^2 / (n_2 - 1)]\}$ [2,4]. In Hrabowski's case, the similarity of the results for the two tests indicates that the use of the pooled t-test was all right. However, if the calculations could have been, and were performed on a computer, it would have been just as easy to use the regular t-test values [2].

Overall, Hrabowski used better and more appropriate statistics in his more recent article. Not only did he use a t-test to compare characteristics shared by both groups, but he also used a matched t-test and an analysis of covariance, ANCOVA, on the two on the two groups. The matched t-test matches subjects in one data set with subjects in the other data set according to key characteristics to try to eliminate the biasing effects of those characteristics. The purpose of the ANCOVA is also to try to eliminate confounding factors, such as gender, high school grade point average, SAT scores, or race, so that, theoretically, the effect is only due to the treatment being analyzed. The covariates in Hrabowski's article included SAT-Math and Verbal scores, High school grade point average, gender, year of college entrance, and the total number of science credits taken during the freshman year. An ANCOVA is an advanced statistical method that is not mentioned in the introductory statistics book that we used to understand the statistics Hrabowski used [3].

In his conclusions in the 1995 article, Hrabowski stated his results in a way more consistent with statistical practice. In the 1977 article, claims were made that extrapolated too far beyond the results of his tests. But in the second article, he stated that " . . . research cannot definitely prove that a given program is the determining cause of observed student outcomes." This indicates that he understands the results from a statistical test can show strong probabilities, but cannot prove a certain hypothesis. One can see that Hrabowski has matured statistically from the 1977 article to the 1995 article. He is using more appropriate statistical tests and his analysis of the results is more consistent.

The little boy who was subjected to extreme racism has achieved more than just “getting the knowledge”. He has grown up to be the President of a nationally known university. Today, in a time where racism is still evident in the US, he has overcome great obstacles and encouraged hundreds of minority students. He is a true inspiration to minorities, mathematicians, and math educators.

References

1. Anderson, Ernest F. and Freeman A. Hrabowski. "Graduate School Success of Black Students from White Colleges and Black Colleges." Journal of Higher Education, Volume 48, Issue 3. (May-June 1977), 294-303.

   Comments: This article is one of the articles analyzed and critiqued. It was the first article found before we found out we could not obtain the Doctoral dissertation.

2. Interviews with Dr. Jill Richie, Professor at Appalachian State University.

   Comments: The interviews were very helpful in the analysis of the statistics found in the articles, as well as the understanding and comprehension of the statistical methods.

3. Kitchens, Larry J. Exploring Statistics: A Modern Introduction to Data Analysis and Inference, 2nd Ed. Pacific Grove, CA: Duxbury Press, 1998.

   Comments: This book provided good explanations for the statistical method used in Hrabowski's articles.

4. Maton, Kenneth I., Freeman A. Hrabowski,. "Enhancing the Success of African-American Students in the Sciences: Freshmen Year Outcomes." School Science and Mathematics, Volume 95, Issue 1. 1995. pp. 19-27.

   Comments: This article provided statistical data and information that was used in the analysis and critique. This article was a good representation of appropriate test statistics.