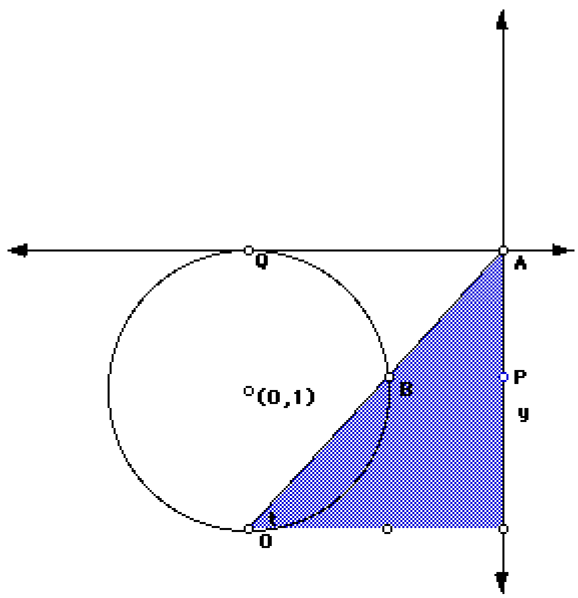
Cartesian Expression



To derive the Cartesian expression to the Witch of Agnesi, we refer to the picture above with certain stipulations, which are given below:

1.) **AQ   OQ**

2.) **AP   AQ**

3.) **AP   BP**

4.) **ΔAQO** is similar to **ΔBPA**

By looking at these triangles above, we can see that the right triangles **ΔAQO** and **ΔABQ** are similar because they share a common acute angle. **ΔQBO** and **ΔAQO** are similar. Now we can get the equation:

$$\frac{\mathbf{AQ}}{\mathbf{BP}} = \frac{\mathbf{QO}}{\mathbf{PA}}$$

If we label **B**=(u,v) and **P**=(x,y), we can use substitution in the previous equation and get:

$$\frac{x}{x-u} = \frac{a}{a-y}$$

We then cross-multiply and get:

$$x(a-y) = a(x-u)$$

After solving the equation for u, the x-coordinate of **B**, we get:

$$u = \frac{xy}{a}$$

By using the Pythagorean Theorem, $a^2 + b^2 = c^2$, where a and b are the sides of a right triangle and c is the hypotenuse, we can derive the following where m is the line parallel to y=2 that goes through the point (0,1):

in Δ**OBK**, $\mathbf{OB}^2 = u^2 + y^2$

in Δ**QMB**, $\mathbf{BQ}^2 = (a-v)^2 + u^2$

in Δ**QOB**, $a^2 = \mathbf{OB}^2 + \mathbf{BQ}^2$

By using all of the above equations above and using substitution, we get:

$$a^2 = (u^2 + y^2) + ((a-y)^2 + u^2)$$

Simplifying the above equation we get $u^2 + y^2 - ay = 0$

We already know that $u = \frac{xy,}{A}$ substitute this into the equation and solve for y and We get the Cartesian expression for the Witch of Agnesi:

$$y = \frac{a^3}{x^2 + a^2}$$