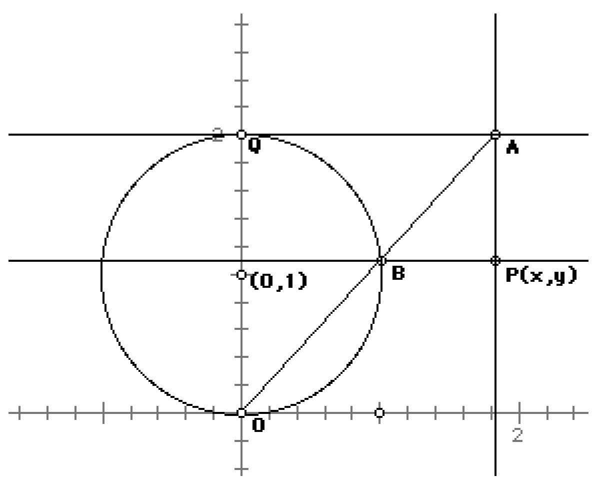
Drawing the Witch of Agnesi

To construct the curve of Agnesi, given by the equation $yx^2=a^2(a-y)$, we begin with a circle centered at (0,1) with radius=1 given by the equation $x^2 + y^2=1$. This corresponds to the curve with a=2, where a is the diameter of the circle. Now draw the line y=2 (or y=a). This line is tangent to the circle and hits the circle at point **Q**. Next, pick any arbitrary point on that line and label it point **A**. Draw a segment from **A** to the bottom of the circle, in this case it is the origin (0,0) and label it point **O**. The point at which the segment **AO** intersects the circle is point **B**.



Next, we must draw a line that is parallel to y=2, that goes through point **B**, and label it line **l**. Draw another line, **m**, which is perpendicular to **l** that goes through point **A**. The point at which line **l** and **m** intersect is point **P**. This is the point that traces the curve when **A** moves along the line y=2. An excellent moving graph that shows how the point **P** traces the curve is found at the following address:

http://xahlee.org/SpecialPlaneCurves_dir/WitchOfAgnesi_dir/witchOfAgnesi.html

Since $y \to 0$ as $x \to \pm\infty$, the x-axis is a horizontal asymptote of the witch. If we move point **A** in the positive direction, the segment **AO** will continue to become parallel to the x-axis, but will never become so because there will always be an angle between segment **AO** and the x-axis. Another observation can be made in that the witch has two inflection points, or points where the graph changes concavity. These are located when the **AOQ** is $\pm\pi/6$.

Several changes occur if the radius of the circle changes. When the radius is increased, the curve becomes steeper. If the radius decreases in size, the curve flattens out. These changes are illustrated below.