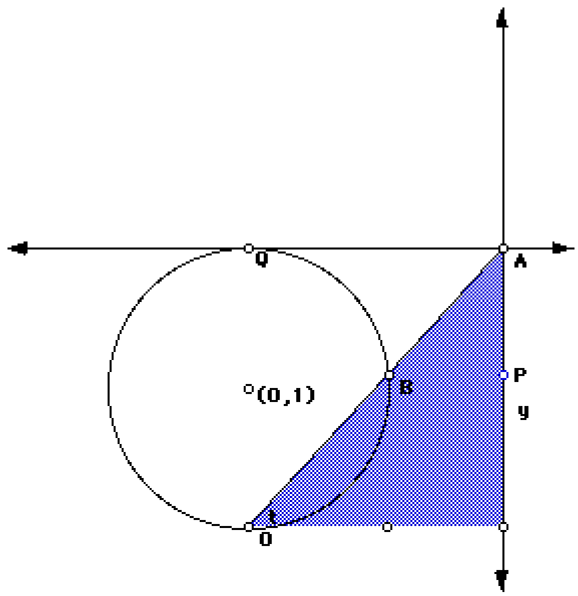
Parametric Equations

To derive the parametric equations for the Witch of Agnesi we should refer to the graph below and assume certain equations.



Assume the following:

1.) x = **AQ**

2.) y = 2 – **AB** sin(t)

3.) **AB * OA** = (**AQ**)

We know that tan(t) = y/x. This also tells us that cot(t)=x/y.

We can solve this for x and get

$$x = y \cot(t).$$

As in our example, y=2, we can conclude that the parametric equation for x is:

$$x = 2 \cot(t)$$

Now that we have the parametric equation for x, we can now the equation for y.

From equation 3 above, we know that

$$\textbf{AB} = \frac{x^2}{\textbf{OA}}$$

Since **OA** is the hypotenuse of the triangle,

$$\textbf{AB}=x\cos(t)$$

From our parametric equation for x, we can use substitution and get

$$\textbf{AB}=2\cot(t)\cos(t)$$

This can be transformed into

$$\textbf{AB}=2(\cos(t)/\sin(t))*\cos(t)$$

By multiplication we get

$$\textbf{AB}=(2\cos^2(t))/\sin(t)$$

If we substitute this into our equation 2, we have

$$y=2-(2\cos^2(t)/\sin(t))*\sin(t)$$

We can now reduce this to

$$y=2-2\cos^2(t)$$

By substituting the trigonometric identity $\cos^2(t)=1-\sin^2(t)$, we get

$$y=2-2(1-\sin^2(t))$$

which reduces to the final parametric equation

$$y=2\sin^2(t).$$

Thus we get the general parametric equations to the witch of Agnesi:

$$x = 2\cot(t)$$
$$y = 2\sin^2(t)$$