The Secant Method

It has been found that the double false position approach to Geometry and Algebra is in actuality the Secant Line Approximation in disguise. Before we look at the Secant Method, let's have a quick review of Double False Position:

Let's look at the example f(x) = x+2x-8+3x-15, and say we wanted this to equal 100. If we let our X_1 = 30, after plugging that into the original equation, we get f(x) = 157. Now, the sum we are looking for is 100; so to find $f(x_1)$ we simply find the difference between 157 and 100. This gives us $f(x_1)$=57. Now we must find an X_2. Let's take the number 21. If we plug this into the original equation we get f(x) = 121. After taking the difference we obtain the value of $f(x_2)$=21. We can then plug these numbers into the double false position formula:

$$X = \frac{(x_2)(f(x_1) - (x_1)(f(x_2))}{f(x_1) - f(x_2)}$$

This process is very similar to the Secant method:

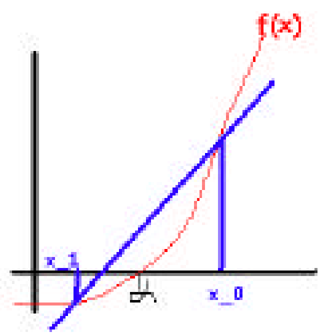
In Kendall Atkinson's "Elementary Numerical Analysis", the Secant method is also referred to as the Newton method. The following is his definition of the Newton method: " This method is based on approximating the graph of y=f(x) with a tangent line and on then using the root of this straight line as an approximation to the root α of f(x). From this perspective, other straight line approximations to y=f(x) would also lead to methods for approximating a root of f(x). One such straight line approximation leads to the secant method."

It is easy to see that double false position and the secant method are going to fall under the same rules and processes because of the use of the word "approximations" in the definitions of each.

To continue with the secant method;

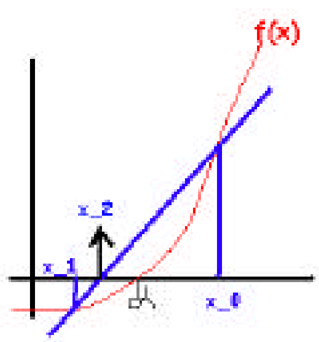
The first thing the student needs to understand is that α is situated on the graph so that it is a root for the curve f(x).

The next thing we would do is assume that two initial guesses to α are known and so we will call them X_0 and X_1.



These guesses are the same as our X_1 and X_2 in the double false position method.

The two points (X_0, f(X_0)) and (X_1, f(X_1)), on the graph of y=f(x), determine a straight line, called a secant line. This line is an approximation to the graph of y= f(x), and its root X_2, is an approximation of α.



To continue, the formula we will derive is the same as the double position formula. This is why we can see that the double position process is really the secant method in disguise.

To derive a formula for x_2, we want to match the slope determined by {(x_0, f(x_0)),(x_1, f(x_1))} with the slope determined by {(x_1, f(x_1)),(x_2, 0)}. This gives us:

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{x - f(x_1)}{x_2 - x_1}$$

Solving for x_2, we get: $x_2 = x_1 - f(x_1)\ \dfrac{x_1 - x_0}{f(x_1) - f(x_0)}$

Having found x_2, we can drop x_0 and use x_1, x_2 as a new set of approximate values for α. This leads to an improved value x_3; and this process can be continued indefinitely. In order to continue indefinitely, we must obtain a general iteration formula:

$$x_{n+1} = x_n - f(x_n)\ \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \quad \textbf{with } n \geq 1$$

THIS IS THE DOUBLE POSITION FORMULA IN DISGUSE!!!!!!

This is called the *secant method*. It is a two-point method, since two approximate values are needed to obtain an improved value. This is the reason we can make the connection between the secant method and the double position method. The both use two unknowns to find the actual value and the formulas used to find this exact value ends up being the exact same formula. This is why we call the double position formula the secant method in disguise!