A Survey of the Mathematics

To understand how to work a false position problem, one must first understand what a false position problem is. The definition stated in the book by Daniel Adams, *“The Scholar’s Arithmetic or Federal Accountant”*, is as follows:

> “Position is a rule which, by false or supposed numbers, taken at please, discovers the true one required. It is of two kinds, Single and Double.
> Single Position is the working with one supposed number, as if it were the true one, to find the true number.

RULE 1: Take any number and perform the same operations with it as are described to be performed in the question
RULE 2: Then say, as the sum of the errors is to the given sum, so is the supposed number to the true one required.”

Single Position

The “single position” method dates back to before 1800 B.C., where problems were found on the Egyptian Rhind Mathematic Papyrus copied by the scribe A’hmose. Two examples from this ancient paper are being used today many school systems to teach proportional thinking and to honor the African contribution.

The best way to understand how a single position problem works is to see an example worked out. A basic single position question would be stated like:

“A quantity and _ of the quantity are added and the sum is 15. What is the quantity?”

The first thing the student (or learner) would want to do is put this question into algebraic notation. To do that, we would simply say:

Let the quantity be represented by “x”. Therefore the question could be stated as x + _ x = 15”

The next thing the student would want to do is to pick a trial solution value. If the question has a fraction somewhere in it, the best trial solution to pick would be the number in the denominator of that fraction. In this case that value would be "4". To get the trial or "false" value, simply plug this "4" into the equation stated above to get:

$$4 + \_ (4) = 4 + 1 = 5$$

However, we can tell that 5 ≠ 15. To get the correct value for "x", we would simply divide 15 by 5 to get 3. This would be called the correction value.

To get the correct solution, we will need to multiply the correction value of 3 to each term in the trial solution value to get:

$$4(3) + 1(3) = 12 + 3 + 15$$

The final answer can be stated as a proportion such as:

$$5: 15 = 4: x$$

In other words, single false position is simply an approach to algebra that was used before our modern methods came about.

Double False Position

Double false position is basically the same as single false position, only that it employs the use of two trial solutions instead of one. If one lets the unknown value be known as "X", then the formula that would be put to use to find this "X" would be:

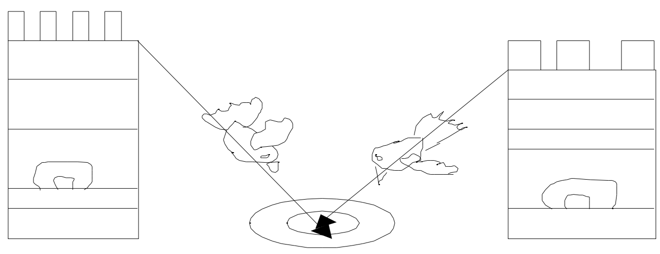
$$X = \frac{(x_2)(f(x_1) - (x_1)(f(x_2)}{f(x_1) - f(x_2)}$$

Just like with single position, the best way to understand these problems is through an example:

> **"Two birds fly from the tops of two towers, at the same time, at equal speed, to a fountain below. They arrive at the same time. How far from the towers is the fountain if**

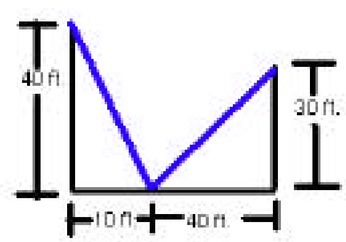
one tower is 40 feet high, the other tower 30 feet high, and they are 50 feet apart at the base? (Find paths of equal length)."

The first thing the students would want to do with a problem like this is to draw a picture. This is very similar to basic geometry problems, and the best way to approach this is through visualization.



The second thing the student would want with this type of problem is find two trial values, x_1 and x_2. To do this, we would have to make some assumptions about the fountain.

1) Assume the fountain is 10 ft. away from the 40 ft. tower.
2) This would make the fountain 40 ft. away from the 30 ft. tower.



We can let the values of 10 be our trial value for x_1. The next thing we need to do is find a value for $f(x_1)$.

3) Since the angles form right triangles at the base of each tower, we can use the Pythagorean theorem to find the values for $f(x_1)$.

For the squared hypotenuse of the triangle formed by the 40 ft. tower, we get:

$$40^2 + 10^2 = 1600 + 100 = 1700$$

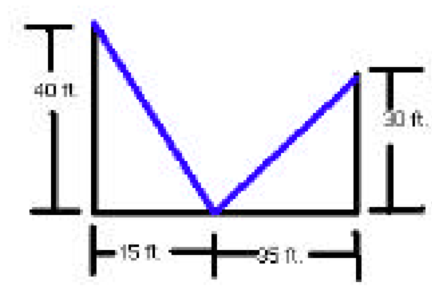
For the squared hypotenuse of the triangle formed by the 30 ft. tower, we get:

$$30^2 + 40^2 = 900 + 1600 = 2500$$

To find the value for $f(x_1)$, we take the difference of these two values:

$$f(x_1) = \mathbf{2500\text{-}1700 = 800}$$

4) **The next thing we need to do is to find an x_2. Let's try a value of 15 ft. for the distance of the fountain from the 40 ft. tower. This would make the fountain 35 ft. away from the 30 ft. tower.**



Using the same process as before, we can use the Pythagorean theorem to find $f(x_2)$.

For the squared hypotenuse of the triangle formed by the 40 ft. tower, we get:

$$40^2 + 15^2 = 1600 + 225 = 1825$$

For the squared hypotenuse of the triangle formed by the 30 ft. tower, we get:

$$30^2 + 35^2 = 900 + 1225 = 2125$$

To find the value of $f(x_2)$, we can take the difference of these two values:

$$f(x_2) = \mathbf{2125\text{-}1825 = 300}$$

WE NOW KNOW ALL THE NEEDED VALUES AND CAN DETERMINE THE VALUE NEEDED FOR "X"

$$X = \frac{(x_2)(f(x_1) - (x_1)(f(x_2)}{f(x_1) - f(x_2)}$$

$$X = \frac{15(800) - 10(300)}{800 - 300} = \frac{12000 - 3000}{500} = \frac{9000}{500} = 18$$

This value of 18 is associated as the distance of the fountain from the 40 foot tower, thus making the distance of the fountain from the 30 ft tower equal to (50-18)=32.

We need to prove that these sums are equal. To do that we would say:

$$40^2 + 18^2 = 1924$$

$$30^2 + 32^2 = 1924$$

These two distances are equal; therefore the birds fly paths of

$$\sqrt{1924} \approx 43.86$$

How Can This Be Used in the Everyday Classroom?

Although false position is no longer used to teach algebraic concepts, its value exceeds its historical interest. Not only does it teach proportional thinking; it also encourages experimentation and problem solving. Selecting a false value and then seeing how it works in the problem results in a more dynamic understanding of how the system operates.