Cox's Work

Cox did work on the difference equation, $aY_{x+1} + bY_x = q(x)$ where a and b are complex numbers that do not equal 0 and whose sums do not equal zero. q(x) is a given polynomial. We will look at the more specific form, $aY_{x+1} + bY_x = (a + b)x^v$. Cox used methods similar to N.E. Norland's methods with studying the equations $Y_{x+1} - Y_x = vx^v$ and $Y_{x+1} + Y_x = 2x^v$. Cox shows that what is called the generalized Euler polynomial is a solution to $aY_{x+1} + b*Y_x = (a + b)x^v$.

Cox uses certain sequences of numbers. One of these sequences, { k } is defined by $a*(k + a + b)^v + b*k^v = 0$ where v = 1, 2, 3,...,c. In the binomial expansion k^v is replaced by k_v. So when v=1 we get

$$a(k + a + b) + bk = 0$$
$$ak + a^2 + ab + bk = 0$$
$$k(a + b) = -a^2 - ab$$
$$k = -a(a+b)/(a+b)$$
$$k= -a$$

This implies that $k_1 = -a$

We run into a little bit of a problem when v=0 we get

$$a(k+a+b)^0 + bk^0 = 0$$
$$a + b = 0$$

This leaves us with no k to solve for. Cox fixes this problem by defining $k_0 = 1$. So we receive the sequence $k_0 = 1$, $k_1 = -a$, $k_2 = a(a - b)$, $k_3 = -a(a^2 - 4ab + b^2)$, ...

Now put k(a + b) = S into $a(k + a + b)^v + bk^v = 0$ and after expansion replace S^r with S_r. So $S_r = k_r /(a+b)^v$. $a(k + a + b)^v + bk^v = 0$ becomes

$$a((a + b)S + (a + b))^v + b((a + b)S)^v = 0$$
$$a((a + b)(S+1))^v + b((a + b)S)^v = 0$$
$$a(a + b)^v(S+1)^v + b (a + b)^v S^v = 0$$
$$a(S+1)^v + b(S^v) = 0$$

Letting q(z) be and polynomial in z we have the symbolic relation

$$a*q(S+1) + b*q(S) = (a+ b)q(0).$$

Now replace q(z) by q(z +x) to get a*q(x + S + a) + b*q(x + S) = (a + b)q(x).

The symbolic expansion of q(x +s) is a solution to the difference equation

$$aY_{x+1} + bY_x = (a + b)q(x).$$