The Order of a Difference Equation and Initial Conditions

.Suppose we wanted to know Y_7. From the rule we can see,

$$Y_7 = Y_6 + Y_5 = 8 + 5 = 13$$

This comes easily because we have already generated the sequence. Pretend now that we do not have the sequence calculated for us and all we have is $Y_{k+2} = Y_{k+1} + Y_k$. Now determining Y_7 becomes rather challenging because we have no way of knowing the values of Y_6 or Y_5. To help with this dilemma we will look back at the generalized difference equation of order n, $Y_{k+n} = F(k, Y_{k+n-1}, Y_{k+n-2}, Y_k)$ in comparison to the Fibonacci equation and provide a couple more definitions.

In the Fibonacci difference equation, n=2 and the order is 2. The order of a difference equation is defined by the difference between the highest and the lowest terms in the equation. The highest term in the Fibonacci equation is Y_{k+2} and the lowest is Y_k . k +2 - k = 2 thus the order is 2. This is how difference equations get their name. The terms in the equation are discrete differences. In general the highest term is k + n and the lowest term is k. k + n - k =n thus n is the order of the generalized equation. The order helps to determine the number of initial conditions needed to lock down a sequence. If the order of the difference equation is n, then n initial conditions are needed to make the sequence unique. The Fibonacci equation needs 2 initial conditions. The first member determined by the equation is Y_2 where k=0. Therefore we need to define Y_1 and Y_0. With the initial conditions $Y_1 = 1$ and $Y_0 = 0$, the Fibonacci sequence is complete and unique.