

### Solutions of Difference Equations

A solution of a difference equations is a function of  $k$  that reduces the difference equation down to an identity. Consider the equation  $Y_{k+1} = F(k, Y_k)$  where  $F(k, Y_k) = 2Y_k$ . Then  $Y_{k+1} = 2Y_k$  which is a first order difference equation can be written as

$$Y_{k+1} - 2Y_k = 0.$$

Substitute  $W(k) = 2^k$  into  $Y_{k+1} - 2Y_k = 0$  and get

$$2^{k+1} - 2 \cdot 2^k = 2 \cdot 2^k - 2 \cdot 2^k = 0$$

Thus  $W(k)$  is a solution to  $Y_{k+1} - 2Y_k = 0$ .

A good way to start finding a solution to a difference equation is to plug in values for  $k$  and look for a pattern. Take the equation  $Y_{k+1} = Y_k + g$  where  $g$  is a constant with the initial condition  $Y_0 = c$  where  $c$  is some constant. Now plug in values for  $k$ .

$$Y_1 = Y_0 + g = c + g.$$

$$Y_2 = Y_1 + g = c + g + g = c + 2g.$$

$$Y_3 = Y_2 + g = c + 2g + g = c + 3g.$$

The pattern emerging in this case is  $Y(k) = c + k \cdot g$ . To test this write  $Y_{k+1} = Y_k + g$  as  $Y_{k+1} - Y_k - g = 0$  and plug in  $c + k \cdot g$  for the values of  $Y$  to get

$$c + (k+1)g - [c + k \cdot g] - g =$$

$$c + g \cdot k + g - c - g \cdot k - g = 0$$

Showing that  $Y(k) = c + k \cdot g$  is a solution to the difference equation.

Do not be fooled by these simple examples. Those of you who have studied differential equations know how challenging they can be to solve assuming they can be solved.

Difference equations are just as difficult if no more so. Refer to the example from Cox's dissertation to see a bit of the tricky nature of difference equations