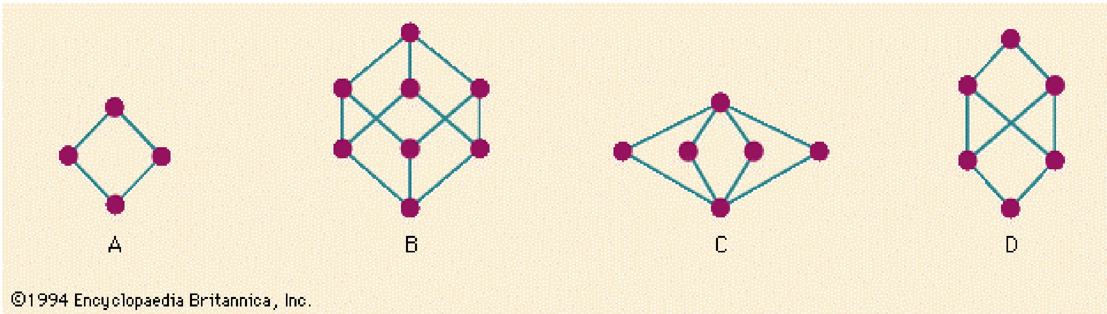
In-depth Mathematics

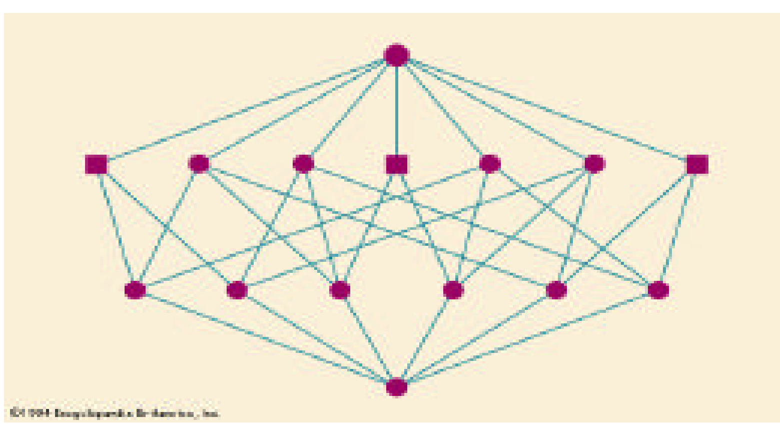
Jonathan Farley's area of research is lattice theory. Lattice Theory is a branch of Abstract Algebra. Lattice Theory also deals with partially ordered sets. First of all, a partially ordered set is a set P with a binary relation, which is denoted as ≤. This is reflexive, which means that $p \leq p$ p P. Also, it is transitive, which means if $p \leq q$ and $q \leq r$, then $p \leq r$. This is anti-symmetric, which means if $p \leq q$ and $q \leq p$, then $p = q$. An example of a partially ordered set is the natural numbers with the usual meaning of ≤. The power set lattice is the set of all subsets of a set with ≤ meaning, "is a subset of" is a partially ordered set. A lattice is a partially ordered set such that any two elements x and y have a least upper bound and a greatest lower bound. The least upper bound is saying there is a smallest element that is bigger than x and y. The greatest lower bound is saying there is a biggest element smaller than both x and y.

According to some definitions of lattice theory, it is the branch of mathematics that deals in precise mathematical language with the relation of different parts of a same whole to each other. The basic concept is that one part, which we will call x, either includes or contains another part, which we will call y. This relation can be written symbolically as x greater than or equal to y. Lattice Theory looks at and analyzes families of subsets X, Y, ... of a given set U under the operation of intersection of sets. This is similar to the way group theory looks at families of symmetry transformations of alpha, beta, ... when combined by taking their composite.

Below are some examples of lattices and non-lattices.



Notice that 5a is the lattice of all subsets of a set of two elements meaning it is two-dimensional, while 5b is a lattice of all subsets of a set of three elements meaning it is three-dimensional. We see that 5c is the lattice of all subgroups of the group G of all six symmetries of an equilateral triangle. The bottom element represents the trivial subgroup, which consists of the group identity. The first three elements above it are the subgroups generated by the reflections in the three altitudes. These are seen in properties of equilateral triangles. The fourth element is a subgroup of rotations of the triangle into itself. The top element is G. Therefore, 5d is not a lattice because the two elements immediately above the bottom element do not have a least upper bound. It is not possible to tell which is the least upper bound.



The picture above represents the partition lattice called Π_4 of all partitions of a set of four elements, which means that by a partition of a set is meant a division of its elements into subsets that do not overlap. The square elements represent the three partitions of the set {a, b, c, d} into two subsets. The two subsets contain two elements. The three partitions are (ab)(cd), (ac)(bd), and (ad)(bc). The circle elements on the same level are representing the four partitions, which are (123)(4), (124)(3), (134)(2), and (234)(1), into two subsets. One of the subsets contains three elements and the other subset contains one element.

Therefore, Jonathan Farley has proved his capabilities in mathematics by researching this topic of lattice theory.